MODERN MINE VALUATION

M. HOWARD BURNHAM, B.Sc.

GRIFFIN'S MINING SERIES

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BY

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WITH 19 ILLUSTRATIONS.

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To My Friends

OF THE SIERRAS AND VELDT,

OF CAMP FIRE AND CAMPAIGN,

OF RIFLE, THEODOLITE, AND GREAT HOPE:

TO THESE BUILDERS OF NATIONS,

BUT EVER VICTIMS TO THEIR OWN TRUST,

This Book is Dedicated.

PREFACE.

Mining is essentially an industry, and its economic justification, like that of any other commercial undertaking, must ultimately depend upon whether it can be made to yield an adequate financial return on the capital sunk in it. But as this depends upon whether the mineral deposits warrant the expenditure necessary for their exploitation, it is evident that the real basis upon which sound mining operations must depend is sound valuation. In the present work it has been the author's object to afford the means of making this valuation by introducing therein the principal considerations governing accuracy. Incidentally, also, it affords a means of ascertaining whether a given valuation is sound and checking the data upon which it is based.

By a large section of the public mining is regarded both as a speculation and as one which is often more than usually "speculative." But the element of chance enters into other industries as well, and may be reduced in mining almost to a minimum, if the valuation be rightly conducted. The author has been at pains to point out to what extent speculation may, in this particular industry, be regarded as legitimate, and to what extent it may degenerate into a mere

"gamble." He has, therefore, written his book as much for the guidance of investors as for that of the engineer, and although he has assumed his readers to be possessed of some knowledge of the subject, he has written it, as far as possible, also from the standpoint of an educated member of the public desirous of ascertaining the nature of the securities he is being invited to acquire.

It may be added that the book is the expression of over twenty years of mining and examination practice in some fifteen different countries, and the main conclusions come to may be summed up as follows:—

(*a*) The most common cause of serious loss of capital in mining enterprises is due to a neglect to apply ordinary business principles and methods in setting the terms of purchase when acquiring properties; and that no sound valuation is possible which ignores these factors. This neglect is thought so radical a defect in our present practice that precedence is given to the discussion of the economic principles which should govern terms of purchase.

(*b*) The next most fruitful source of loss is the failure to set appropriate values on blocks of ore in different stages of development. In other words, to give practical recognition to the manifest fact that the risk varies with the uncertainty. A method and some twenty tables for such allowances are offered both for application to blocks of ore and for share valuation. The tables incorporate not only the actuarian calcula-

tions incidental to the life, but to the delay and risk incidental to realising upon each block of ore.

(*c*) It is also thought that a large proportion of mining failures would have been avoided had engineers felt obliged not only to collect but to submit the fullest field data with their reports. This refers more especially to a graphical representation of sampled faces, giving as well plotted and detailed assay values. A system of such graphical presentation is given, with a closed transverse method of checking sample section measurements.

(*d*) As part of a consulting engineer's work calls for the valuation of mining shares, ten tables have been worked out to meet the varying rates for risk from 0 per cent. to 20 per cent., as well as life, which the engineer may wish to assign.

(*e*) Consulting engineers having constantly to recommend or depend upon others for work abroad, they are frequently at a loss to know if such are adequately grounded in the basic principles governing the selection and setting out of field data. Hence the book should be found a useful guide to subordinates as setting out definitely what may be expected of them. Naturally, a knowledge of technical detail is assumed, such, for example, as is set out in "Mine Sampling."*

The book may thus be regarded as a companion volume to Lawn's *Mine Accounts,*† although the sub-

* "Mine Sampling," by T. A. Rickard, *Mining Magazine*, London.

† *Mine Accounts and Mining Book-keeping*, by Prof. J. Gunson Lawn. C. Griffin & Co., Ltd., London.

ject with which it deals is valuation, pure and simple. A number of diagrams and illustrations have been introduced, and numerous tables given to simplify calculations and check results. The author hopes that the work will prove useful to those for whom it has been designed, and not less so because he has, in dealing with his subject, covered a somewhat wider ground and introduced considerations of a more general nature than are generally found in works of this description.

M. H. B.

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ERRATA.

Page 34, line 15, *for* (18) read (17).
,, 52, line 9, *for* k *read* k'.
,, 60, Table III., col. 6, *for* Feet *read* Dollar-feet.
,, 66, line 7 from bottom, *for* 20 feet *read* 26 feet.
,, 70, line 6, *for* three *read* two.
,, 96, line 5, *for* Fig. 14 *read* Fig. 13.
,, 107, line 15, *for* L *read* H-g.
,, 109, line 1, *for* f-g-h *read* f-g-H.
2, *for* g-h *read* g-H.
3, *for* O′-H *read* O′-H′.
,, 110, line 3, *delete* "green."
,, 116, line 4 from bottom, *for* $\frac{\cdot 03}{(1 - \cdot 03) - 1}$ *read* $\frac{\cdot 03}{(1 + \cdot 03)^n - 1}$.
,, 116, line 5 from bottom, *for* $\frac{r'}{R - 1}$ *read* $\frac{r'}{R^n - 1}$.
,, 151, line 7 from bottom, *for* $S - (1 + s)$ *read* $S = (1 + s)$.
,, 158, line 23, *for* £800,000 *read* £80,000.

MODERN MINE VALUATION.

INTRODUCTION.

PRELIMINARY CONSIDERATIONS.

The practice and principles of valuation, as generally met with in the English-speaking mining world, often leave much to be desired, and are open to a good many criticisms, despite the fact that they embody many well-established conventions.

Many mining projects which may be called pure speculation have much to be urged in their favour, providing the enterprise be plainly labelled, and the adventurers given an honest run for their money, which usually means that all capital outlay goes to development. At the same time, the warping of facts and the distorted deduction which so often deliberately confuses the two stages in mining enterprises, may be branded as contemptible, to put it mildly.

Especially with reference to mines in the Company stage, it is held that the collection and certification of data forming the basis of valuation is altogether inadequate in the case of nine out of ten mining reports, even where large sums of money are involved.

If one but compare the time and detail, the collection of and compilation from signed vouchers which lie behind the balance sheets of large commercial enterprises, with those forming the basis of mine valuations of similar magnitude, the impartial critic can hardly fail to be struck by the difference so often displayed, and the apparent regard for what, in the first instance, are habitually regarded as guiding principles. In other words, it is held that, while the true inwardness of a report, where the mine has reached the Company stage, can and should be set out on a single sheet of paper—in fact, nearly as concisely as a balance sheet—yet the data forming the basis should be collected and vouched for at each successive step in a manner identical in principle with that obtaining in ordinary commercial valuations.

For example, a sampler should sign each vein section sampled by him, just as the surveyor signs a plan, or a chemist an assay certificate. Again, the field data, as well as the tabulated analyses of these results, should be as clearly arranged as the time-books and store-sheets of a manufacturing enterprise or producing mine. In a word, it may safely be said that, in point of certified accuracy, mine valuation, as generally obtaining, is wofully behind the usual standard of commercial undertakings, both in principle and practice.

While tabular forms for data and calculations are found further on, they are simply those used by the writer in ordinary work, and should, of course, be modified to meet varying local conditions and the convenience of those employing them.

In the valuation of mines in the Company stage, it is held that two essentially different features must be clearly set out, one being the appraisal of the blocked ore bodies or probably profitable portion of the deposit, the other being, in the case of veins or tabular deposits, the estimate of shoot dimensions in economic units, the latter as a bearing upon shoot extension in depth. It should hardly be necessary to add that the last may not be the only factors in such calculation; in fact, when a vein cuts different types of country rock, or neighbouring veins show, say, zinc and coincident reduction of precious metal values, these, instead of the shoot dimension, may be the dominant consideration.

While the present work deals with elements only, it is not meant for beginners, nor those to whom the commoner problems of mining are wholly unknown. Much is taken for granted, both with regard to the technique of the subject and to those phases of human nature especially evident in more speculative undertakings. The aim is to touch upon features of what may be called Investment, as distinct from Speculative Mining, and the writer would again emphasise the great difference between the two in spite of the fact that they must merge one into the other. This should not, however, be read as a reflection upon the latter. Many forget that the modern small Syndicate for opening up mines in Africa and far lands is but the present-day descendant of our forefathers' "Adventurers' Companies," who sailed the seas, looted the Spaniard, and lost life and treasure like men. How often has one to uncover, as it were, before the spirit shown in the winding-up meetings

of these Syndicates and Companies, and one's faith in the race grows with such evidences of its gameness. *Apropos* of this healthy outlook, one recalls regretfully the great lodes, virgin to work and full of promise which one sees occasionally, and upon which one would gladly advise some small expenditure if only sure that it would go to actual development. Too often prominent engineers lack the courage to advise outlay save on developed properties, and the lesser man, copying the others, would outdo them in an affected caution.

The shrewd student of modern economic development will not have failed to notice the over-emphasis on mechanical, as distinct from administrative, organisation. Similarly our theories and practice of Mine Sampling, while covering a wide field, have apparently failed to develop a practical expedient tending to act as an automatic check upon the vagaries or indifference of the sampler. A system devised by the writer to meet this difficulty is offered, though, bearing in mind the large amount of literature devoted to Mine Sampling, it would seem well to do this with a becoming show of diffidence, more especially as an effort will be made to present other features of the work as well in a new light.

A thorough study of even the elements of the subject must be incomplete without a discussion of the principles underlying "Data and Deduction therefrom": also, "Probability and its Applications." These, however, must be left untouched, fascinating and important though they be.

It may be mentioned that the writer's system of

sampling, as dealt with later, was devised some twelve years ago on the Rand, to meet the needs of a 30-foot stope, and while the principles here set out are of nearly universal application, it should be borne in mind that every mine demands variations in sampling practice. The above system, for instance, is of special value when dealing with large veins of banded pay and final examinations, though the cost and time might possibly preclude its use for preliminary work or the valuation of small veins.

Most of the tables at the end of the book were compiled some ten years ago, to meet the author's needs, and while some of them will be found to be old friends dressed in modern garb, this very change may be an agreeable innovation when compared with the cumbrous terminology usually employed.

To those who find interest only in such theory as has had application in practice, it may be mentioned that the ideas expressed are the outcome of a mine examination resulting in the single cash payment of nearly a million pounds sterling, during the course of which work the writer found himself at issue with several well-known engineers, it being found that the views held were so widely divergent as to involve, not only the elements of block valuation, but the principles of economics.

It must be clearly understood that the whole argument is based on the general underlying premise that mining is to be regarded as an industry, and that the status of the engineer is that of a specialist, intermediate between the scientist and financier—*i.e.*, his claim to

social utility must rest as much upon his knowledge of economics as of natural science.

Divisions of the Subject.—Mining valuation may conveniently be divided into the study of reserves or assets, and the more variable quantities or possibilities, and while an attempt will be made to deal with the former somewhat fully, the latter will only be touched upon sufficiently to set out differences. As will be seen later, it is held that mining reserves may be valued in a more rational manner than that now in general use, and a method is offered which, if accepted, should tend to the development of the industry. An apology is offered engineers of more modern views for the statement of self-evident facts and perhaps unnecessary elaboration of elementary premises, the excuse being that a somewhat intimate knowledge of both English and American conditions appears to indicate the advisability of dealing rather fully with the subject, even at the risk of redundancy.

Because of the demand for rifle as well as theodolite, for courage as well as technique, for travel as well as toil, for languages as well as science, probably no profession calls more insistently for accuracy, pluck, ability, and sound general education than that of mining. Because, however, of the facility with which ineptitude and fraud may be covered by a puerile parade of technicalities, or hidden in distant mountains, the profession is too often represented in the public eye by men of inferior attainments.

FUNDAMENTAL PREMISES.

Premise I. — *A sense of economic proportion is essential to effective examination work.*

It would be idle to ignore the fact that a large portion of the examining engineer's energy, particularly while he is "winning his spurs," must be spent in seeking graceful and effective compromises between honesty and expediency, but the later developed sense of economic proportion, or related civic proportion, ultimately urges expediency to accuracy beyond arithmetic, even to that phase of calculation which deals with the ratio of dividends to capital.

Reporting Engineers may well admit that in the body politic we are but hucksters; vendors of wisdom seeking sale for our wares. Our fragile commodity may be a work of art; a creation born of inspiration, polished by travel, tempered by experience, and yet be unsaleable. To fetch a good price it must take the public fancy, and yet be approved by those clever connoisseurs and shrewd purchasers of our products known as Financiers.

The latter claim with much feeling and some justice that the engineer is not a business man, meaning thereby that they lack a sense of economic proportion. For example, when sent to examine a property upon which an option is held, his sole idea seems to be to approve, or otherwise, of the purchase. If his report condemn at the price, he in nine cases out of ten makes no effort to secure more reasonable terms; if the ore "blocked out" be insufficient to justify the outlay, he would seem to consider it inconsistent with his professional dignity to negotiate a new option with time in which to develop.

Again, in the case of a going concern, overlooking the

limitations of geological knowledge, he voices his fears as if they were facts. In other words, he at times inflicts an immediate market loss on shareholders through airing his geological postulates of mere possibilities, thereby seeking to display technical ingenuity, or perhaps having regard to a fear of rigid professional criticism.

Still, again, there is a notion prevalent with the more honest engineers that they and their offices only are essential to the industry ; that the men who at trouble, expense, and the exercise of sound judgment—the result perhaps of years of experience and loss—get together the capital wherewith to operate, are necessarily parasites.

Here, again, we have the same narrow vision which, in the field, often operates disastrously by studying too limited a geological horizon ; which confines itself to pure or stereotyped sampling ; which ignores the habit of the ore shoots in adjoining mines as a means of gauging probabilities touching the one in question.

Again, the most puritanical of engineers must have admitted during the latter "90's" the flotational value of an eastern extension on the Rand covered by the coal measures, even though the data for definite valuation had been wanting.

As mentioned elsewhere, honest and well-informed speculation is as legitimate an enterprise as investment, and a sense of economic proportion would demand, not only a careful consideration of features governing the risk-rate, but a well-balanced knowledge of these factors which affect popular judgment, and hence facilitate the often arduous task of securing capital, necessary alike to clean and unclean promotion.

As before mentioned, it is held that the mining engineer is more inseparably connected with the larger financial aspects of his operations than are the other professions referred to with theirs. If this view be well taken, it would seem fair to examine his pretentions by the light of his development of what might be called "Economic Units."

Units.—If an Electrical Engineer were asked the practical possibilities of a stream of any magnitude, he would at once state it as so many thousand kilowatts in the season of a minimum flow.

It will be noticed that here is an expression of uncertainty; and acknowledgment that Nature may step in and render calculations abortive. Yet this does not affect his development and use of comprehensive units.

If, on the other hand, the mining engineer were asked to compare a vein in cobalt with a reef on the Rand, he would be at a complete loss to find a common measure for the possibilities of each without long and special calculation; he would appear to be at the mercy of various untoward contingencies incidental to ore bodies. Yet the civil engineer, again, develops his own units; calculates his stresses and strains to multiply by a factor of safety of two, four, or ten, accepting without comment the fact that chance too may prove his calculations wrong, and cast his edifice to the ground.*

But the energy of many mining engineers seems to find its chief expression in a study of the cost of ore

* It may have escaped the attention of younger students that what appears to be an unnecessary refinement is *really scientific, even though inexact,* as it is the effort to reduce the effects of the "personal equation."

dressing or of metallurgical operations, which, however important to the mining engineer, cannot be of the first magnitude in his ultimate views, which should be mainly directed on the *source of income.*

What is needed is a common measure of practical possibility or probability based on the data at the time available ; gauging the potentialities of the stream, as it were, by the measurements taken, though allowing for dry seasons ; approximately appraising shoots, beds, or chambers, by a common factor which must be economic not geological. Such considerations have led to the invention of such numerical aids to valuation as value-feet, cost-loss feet (or combination factor), profit-feet, and shoot-feet. In a general way this latter may be said to be derived from average profit-feet and aggregate shoot-length.

Thus harmless diversion may be derived by plotting value-feet minus combination - factor or cost-loss-feet at each point sampled on the deepest full shoot level of the mine, although it may be seen that this curve of profit-feet has at times an uncomfortable habit of dropping below zero, a planimeter will give the shoot-feet (see Figs. *5a* and *5b*). Many will refuse dealings with an innovation hiding behind its simplicity so many demands upon exactitude, and so given to uncovering to nakedness mining in waste places ; but the shrewder Daniels of the potential directorates will recognise the value of shoot-feet in classifying and relegating, while it often spells more than "ore blocked out."

Combination - Factor.— The term "Combination-

Factor" is meant to cover loss in as well as cost of treating each ton of ore. For balance-sheet purposes—*i.e.*, taking the mine as an asset—the features of which are being studied, whether the values be left in the ore, fills, sorting dumps, or sands; whether it be expended in mining or milling, the result is much the same. It is, of course, evident that in mine development stoping costs and losses will vary with the position, thickness, parting slips, and character of the ore bodies and blocks. Hence the combination-factor will vary; its weighting will call for experience and intelligent study; it will not be exact, but more so than the expression of an average cost applied to all stopes. One ventures to add that its possibilities grow upon one with use.

Premise II.—*Modern reporting practice calls for a full presentation of data, calculations, and deductions therefrom.*

Balance-Sheet and other Schools.—As elsewhere mentioned, there may be said to be two schools with regard to mining reports; one would seem to aim at expressing a guarded opinion only, eschewing the publication of data, and thereby making sure of successful evasion in case of criticism, while protecting itself against the detection of slovenly work. The other, the balance-sheet school, is largely the outcome of modern scientific teaching, and tends to regard the presentation of data and calculations as important as the expression of opinion. The rise of the consulting engineer with his field assistants and managers is also responsible for this more modern demand with respect to reports. How often, through want of adequate detail, one has to reject another's opinion of a property where, had full data been submitted,

the point of difference might have been allowed for, and an adjustment made leading to promising adventure.

Field Notes.—To the writer's mind field notes belong to the one paying for the report, and should have the same value assigned them as that given the field books of a railway survey. Further, one may and should demand a clear statement of such data and deductions therefrom, as is called for by Premises III. to X., thereby enabling another, not only to resample any section in the mine itself, but to check each step in the subsequent calculations.

The point specially emphasised is, that from the first stage of physical measurements at school, on to the development of empirical formula in workshop or laboratory, the engineer, though subordinate to manager, is ever calculating, studying, deducing from or submitting data collected. His presentation of costs and losses in many annual reports are monuments, not only to his courage with regard to criticism, but to the thoroughness of his belief in the justice of the demands upon him for detail. Moreover, when the practical sequence of examination work of any magnitude is considered, with which we are now principally dealing, one has to admit the succession of engineers from junior to senior and to super-senior before favourable finality is reached. Such being the case, unless detailed reports be forthcoming, apart from an expression of pure opinion, the labour, knowledge, and experience of each would be practically unavailable for his successor. The only legitimate ground for disagreement concerning this premise is with regard to what may be considered a *full* representation

of data. Naturally, that required of assistants or colleagues by the consulting engineer will depend upon the latter's training, standards, and personal equation. Nevertheless, the current literature and proceedings of technical societies furnish a rough guide to what may be considered good practice.

The fulness of detail formerly shown by reports on Rand properties is perhaps a high water-mark of excellence in this respect, though the data set out in recent examinations of some of the great American copper properties leave little to be desired.

The great stumbling-block to conciseness, particularly with regard to final deductions, is the disinclination on the part of engineers to face the truth concerning their own responsibility in the matter of capital outlay. In their reports some touch on "gross value," yet seek to slur over the "gross"; others play with "annual profit": some few show an academic interest in the return of the capital embarked. But how many have the courage to start with the assumption that the essential to sound investment is the return of both capital and interest? When to this be added an interest commensurate with risk, as demanded in practice by at least two eminent engineers known to the writer, one meets a spirit and exactitude never to be expected, save perchance under the lime-light of technical analysis.

Mine Reserves. — Mine reserves in the sense used hereafter may be defined as bodies or blocks of ore or mineral, concerning the value of which sufficient physical data is available to warrant an immediate cash purchase.

It will be noticed that this definition implies a common

concept concerning the nature both of value and data, something that does not exist, but to the establishment of which it is hoped the present effort will contribute.

Value as used here has the meaning usually attached to it when represented in monetary units, as shown by Premise III. With regard to both the sufficiency and interpretation of data, even apart from geological ideas, the greatest difference is noticeable, due, according to the writer's belief, to the over-emphasis of technique or its equivalent—the under-weighting or economic factors.

Premise III. may be stated as :—

Premise III.—*Any investment implies the expectation of a return of the original capital,* 3 *per cent. annually to represent the rate received by investments conceived to involve a minimum risk, and a further rate of interest, commensurate with the risk, and counting from the date when the investment was made.*

Corollary (*a*).—When dividends are deferred or suspended, those received must make good the loss due to such deferrence or suspension of interest.

Corollary (*b*).—As the interest during the deferred period may be regarded as a further investment of capital in the undertaking, interest on this at the risk-rate is to be expected ; in other words, allowances for loss due to deferrence of dividends should regard the interest as compounding during the deferred period.*

Corollary (*c*).—As blocks of ore in a mine vary in tonnage and value, and data (or number of sample sections) available, as well as time of exhaustion,

* See Appendix A, which deals with the systems of computing the value of deferred annuities.

estimates of their present value involve a consideration of "life," risk, and deferrence for each.

As outlined in Premise III., the return of capital as well as interest is a fundamental concept of industrial economy, hence any undertaking which jeopardises the return of either involves risk.

While we cannot draw a sharp line between speculation and investment, we may say that, in general, the one seeks increase in market or capital value, and the other a satisfactory return of interest; the one largely looks to the variable human element for advantageous changes and the other to the constant demand for livelihood or its equivalent as insuring interest on capital so embarked.

In the long run the demand for a higher rate from those projects considered to entail greater risk is reflected in its equivalent market value, so we can say that, at least in public estimation, the risk varies directly with the rate expected; this, of course, assumes a free market, and selling, as distinct from nominal or exchange quoted value.

The difference between investment and speculation may be noted by the varying weight given the data submitted, having its corresponding expression in the rates of interest demanded. At times the public will feel so certain of the data concerning a man's integrity and ability, that little more than his name is necessary to give or maintain market value, at least for a time. Again, the most complete detail may be ignored if the names attached be unknown or unsatisfactory. In such cases one sees at times the economic significance of a name for commercial honesty.

The two features of a going mining concern—quotations and dividends—based on the above elements, have to be carefully borne in mind, and while the factors affecting human optimism, aberration, and hysteria. hence market fluctuation are too complex to be dealt with, they, more often than real value, dominate the outward situation ; seldom, however, over long periods.

It would seem more than probable that the possibilities attached to the finding of bonanza—hence the hope of enrichment independent of market manipulation—are at the root of the attractiveness of mining ventures ; this very legitimate phase of speculation appertains essentially to the engineer's work, as distinct from that of the financier.

The universal demand for a higher rate from those undertakings involving the greater risk is of such universal acceptation that it would seem to form a common premise ; it is, of course, based on experience such as underlies the probability curves,* upon which rest nearly all scientific measurement.

The Insurance Principle.—The demand then of an investment acknowledged to involve risk are :—

The return of the capital embarked ;

A rate of annual interest, such as is judged by non-hazardous investments as, say, 3 per cent. ;

A further rate of annual interest ; this to be a function of the risk.

The first two requirements would seem to call for no discussion, at least of their theoretical aspect ; the last,

* See Formulæ (17) and (18).

however, implies the recognition of the insurance principle, though applied inversely. For instance, an Insurance Company receives small annual payments over a series of years, which, in the case of the man of average life, more than repays that paid out at his death. Yet the insurance might have to be paid out immediately after the receipt of the first premium, which would in itself have been a bad investment or risk. And yet when a large number of such risks are undertaken, a business is built up which is ordinarily considered to be of the soundest nature. In the case of speculation, the tendency and the expectation of enlightened people are a number of small losses succeeded by a great gain. Naturally the man in the street expects to keep clear of the losses, and so to time or chose his adventure as to reap during the fat years, and escape the lean ones. This is somewhat analogous to the man who only insures when ailing. From this it will be seen that theoretically a considerable number of adventures must be made in order to constitute sound practice, a fact ordinarily lost sight of.

The above general demand for a higher rate of interest from mere speculative projects is a curious case of unconscious cerebration regarding probability, even where no intention exists of embarking upon further ventures of a similar nature.

It may be contended that a higher rate demanded of an investment will not protect against loss in case of failure before redemption of capital and interest thereon. This is true, but if this high rate be expressed in terms of lower present value, the amount risked is less; or,

expressed differently, a high rate of return will pay off a larger part of the capital risked, should failure occur before all was repaid.

The unconscious application of the risk-rate principle is shown by the issues of the City of New York, which, while it cannot borrow at 3 per cent., can do so at 4 per cent. ; evidently in case of repudiation, the extra per cent. is no protection against loss, but it reduces it were any dividends received. Again, the City of Rio de Janeiro cannot borrow at the rate of 4 per cent., but should it offer 10 per cent. large sums would be forthcoming, even though the conditions affecting the risk were unchanged.

From the above it will be seen that the essence of financial practice is to demand a rate which varies with the risk, whether this be soundly estimated or not ; in other words, there is an unconscious application of the principle of probabilities and its series of investments, even though the dominant thought is profit and a single venture.

Expressed otherwise, we will not buy Honduras 6 per cent. paper at 90, but would at 40. Why? There would be a loss in either case were repudiation to take place within a few years. Simply because with the higher rate a greater part of the capital is returned, giving further opportunity to re-adventure. The financier will from daily practice have little difficulty in accepting the principle of the demand for a rate varying with the risk, but perhaps, because of the unhappy lack of sense of economic proportion on the part of engineers, the true significance of the principle would seem to have escaped them.

The foregoing may be expressed as—

Formula (1). $$D = r' + r'' + r'''.$$

D standing for yearly dividends in percentage of capital, and assumes a uniform rate of dividends throughout the life of the mine ; r' is 3 per cent. ; r'' the rate set aside to redeem capital : r''' is the risk-rate.

The idea at the back of Formula (1) is not put forward as a novelty, but it has heretofore, in the writer's opinion, been most insufficiently weighted ; it is to both the financier and reporting engineer what Newton's Principia are to the mechanical man ; it is talked of, acted upon, and unknown. The possibilities of its application would appear to be unsuspected even by many thinking men ; it is the touchstone of speculative morality.

As will have been gathered, the basal idea is that whatever the rate of dividend received, it may be divided in such manner that each part will serve a special purpose. This seems the simplest way of regarding and treating the subject of capital redemption and deferred annuities. It may be as well to again point out that the applications of such calculations and tables therefrom are not meant as a certain determination of value, but a simple mathematical presentation of principles, having in view the practical recognition of the insurance principle, hence the necessity of isolating those factors in the formula representing risk, with which one is here principally concerned. For instance, redemption is taken at 3 per cent., because even 4 per cent. represents a greater risk, as is indicated by the purchase of Government paper to

bear only 3 per cent., the great desideratum of the latter being security.

Formula (2). Let $C = \frac{D}{r}$ for (n) years,

the assumption being made that D is a uniform dividend, which is to run (n) years, the estimated life of block, or number of annual payments ; C stands for the capital or present value ; and r for the annual rate of dividends, but

Formula (2*a*). $r = r' + r'' + r''' + r''''$.

Where r' = the rate paid by Government paper here taken at 3 per cent. per annum ;

r'' = the rate which it would be necessary to receive for (n) years in order to redeem capital C (*i.e.*, sinking fund or $\frac{r'}{(R^n - 1)}$. See p. 24.

r''' = a factor of safety, so to speak, which if set aside would be sufficient to repay the loss due to unforeseeable events, providing a number of similar ventures were made—*i.e.*, r''' is an insurance allowance.

r'''' = the rate set aside which during (n) years would make good the interest when dividends are delayed (d) years. As shown on p. 24, this is—

$$(S^d - 1)(r'' + s),$$

where $s = (r' + r''')$ and $S = (s + 1)$.

From the above one may write—

Formula (3). $$C = \frac{D}{r' + r'' + r''' + r''''}.$$

In the case of a going concern paying dividends at the time of purchase, $r'''' = 0$, as would also r'' and r''' in the case of a good Government security selling at par.

If the dividends are expressed in percentages of capital, then the value of C will be in decimals, hence may be used with pounds, dollars, or other monetary units.

The variables, represented in Formula (3) by D, r''', n, and d are given in the following tables, some of which are arranged to give D different values expressed in percentages, while in Tables xxvi. to xxxix. it has a value of 1, as in an annuity of £1.

It is believed that they will be bound to conform more closely to modern ideas, as well as to advance the use of financial analyses.

It will be noted, when considering the tables, that the risk rate (r''') is taken as the rate paid by, or demanded of, the investment beyond r'' the sinking fund rate, and r' (or 3 per cent.). This is based upon the contention that all business practice of to-day leads us to as certainly count upon 3 per cent. interest as upon capital return, hence only the portion of the rate above these two is available for insurance—*i.e.*, to cover risk.

Delayed Dividends.*—The ordinary treatment of the subject of deferred dividends, where two rates of interest obtain, seems to take into consideration the risk to capital only, disregarding the claim of interest on that capital for risk-rate during the deferred period.

* In keeping with the effort to employ general business concepts only, delay means the time by which the receipt of dividends exceeds one year, as no one expects a dividend under one year from date of investment (neglecting semi-annual dividends). See also Appendix A.

The commonly accepted reason for a higher rate being asked of one investment than another is the greater risk in the one, and that the difference between the two rates is to be regarded as an insurance. If we endorse this view, then from the very nature of insurance, when risks are spread over several investments, the difference between the rates obtained and that had from, say, Consols is set aside to make good the losses in one or other of the undertakings; hence the difference between the rates is essentially to *make good interest loss as well as loss of capital.*

If this be ceded, then the rate of interest during the deferred period upon the interest which should be considered as accruing must be such as will recoup any loss of interest; in other words, the interest during the deferred period should be compounded at the high rate, inasmuch as both capital and interest during this period are essentially embarked in the undertaking itself; in fact, from the nature of insurance the two are identical.

It is ceded as an axiom that the owner of a deferred annuity must be in the same position at the end of the period of investment as he would have been had there been no deferred period. On this basis only by a risk-rate on the deferred high rate of interest would he be in the position sought.

It will be noticed that this essential difference of view from that of several accepted authorities lies in the definition of insurance; also that its numerical recognition (r''') in mining calculations calls for higher rates than are ordinarily considered.

One may here stop to wonder if the fact is grasped that it is not stated that money may be invested in practice and compounded at a risk rate of, say, 20 per cent., such as some block calculations would entail in theory, in order to arrive at sound estimates of value.

Returning to Formula (3)—

Formula (4). $C = \frac{1}{s + r'' + r''''}$ = the present value,*

where the (s) now stands for ($r' + r'''$), or the rate we expect to receive as interest, dividends being unity.

It follows from the above that, when r'''' is set aside for (n) years and accumulates at r' interest, we should have

$$\left(r'''' \frac{(R^n - 1)}{r'}\right).\dagger$$

This must make good the interest and risks during the deferred period (d), treating this accumulation as new capital, which must have interest paid on it, at r''' rate for (n) years, though compounding at r', and not at s, as was found necessary during the deferred period.

From the above reasoning we can write—

Formula (5).

$$r'''' \frac{(R^n - 1)}{r'} = \left[(s) \frac{(S^d - 1)}{s}\right] + \left[(s) \frac{(S^d - 1)}{s} (s) \frac{(R^n - 1)}{r'}\right],$$

where $S = (s + 1)$.

* This is sometimes known as the Hosvold formula; this and others will be more fully discussed in the Appendix. It is hoped that the more intelligent reader will realise that the aim is not to insist on one formula, but to urge that the deferrence principle must find practical recognition.

† See Formula (10).

Formula (6).

$$r'''' = (S^d - 1)(r'' + s) = (S^d - 1) r'' + (S^d - 1) s,$$

i.e., r'''' must redeem the new capital $(S^d - 1)$ and pay interest on same at the $(r' + r''')$ rate.

If the value of r'''' be substituted in Formula (4), we have—

Formula (7). $$C = \frac{1}{(s + r'') + (S^d - 1)(s + r'')}.$$

Formula (8). $$= \frac{1}{S^d (s + r'')} = \frac{1}{(1 + s)^d r'' + (1 + s)^d s}.$$

Formula (9). $$= \frac{1}{[(1 + r' + r''')^d][r' + r'' + r''']}.$$

From any algebra setting out of the principles of annuities, the amount C at the end of n years is :—

Formula (10). $$C = \frac{D(R^n - 1)}{R - 1} = \frac{D(R^n - 1)}{r'}.$$

Where C = capital to be redeemed by the annual dividends received ;

D = annual dividends received, in per cent. of capital ;

r' = Consol rate, also the rate realised upon the investment of the annuity ;

R = $(1 + r')$;

n = life of mine in years.

But by Formula (1)—

Formula (11). $D = r' + r'' + r'''$; and as the rate necessary to redeem unit capital is alone considered ;

$r' = 0 = r'''$, hence

$D = r''$ and (10) becomes

Formula (12). $1 = \dfrac{r''(R^n - 1)}{r'}$.

Formula (13). $r'' = \dfrac{r'}{(R^n - 1)}$ $\left\{\begin{array}{l}\text{when C is taken at}\\ \text{unity.}\end{array}\right.$

Formula (13a). $r'' = \dfrac{r'}{\left(1 + \dfrac{r'}{m}\right)^{mn} - 1}$ $\left\{\begin{array}{l}\text{where interest is}\\ \text{received and re-}\\ \text{invested } m \text{ times}\\ \text{per year.}\end{array}\right.$

Hence our basic formula for block calculation is—

Formula (14).

$$\left.\begin{array}{l}\text{C (the present}\\ \text{value)}\end{array}\right\} = \frac{1}{(1+s)^d \dfrac{r'}{R^n - 1} + (1+s)^d s}\text{, or}$$

Formula (15). $$C = \frac{1}{(1 + r' + r''')^d \left(r' + r''' + \dfrac{r'}{R^n - 1}\right)}\text{, or}$$

Formula (16). $$C = \frac{1}{(1+s)^d \left(s + \dfrac{r'}{R^n - 1}\right)}.$$

External Factors effecting the Risk-Rate.—While the application of the risk-rate principle to that phase of valuation with which we specially deal is indicated by corollary (c) of Premise III., both its use and developments call for further discussion: for example, the value to be given to the risk-rate (r''') in order to allow for variation in the selling value of other metals than gold, as well as the profit per ton in the ore, is worth more than a passing thought, but will be only touched on here. Sometimes we see that copper shares, for instance, are purchased at such figures as to bring the

interest paid by a company producing copper at £50 per ton to the same basis as one producing at £30 when the selling price of copper is, say, £60; this in spite of the fact that a fall of £10 in the price obtained for the metal will cut off dividends entirely from the one and only reduce those of the other.

As a matter of fact, while an exact adjustment of the risk-rate (r'''), whereby to meet the vagaries of the metal market, would be impossible, it will appear that the probability of a cessation or reduction of dividends would follow the ordinary probability law expressed by

Formula (17). $y = \frac{k}{x^n}$,* or possibly by

Formula (18). $y = \frac{k}{n^x}$,*

where y is the probability of loss, x the difference between the cost of production of the metal and its mean selling price, while n and k are constants as determined by experience.

It is a source of constant wonder that people invest in copper or silver ventures on the same basis as gold—*i.e.*, that they accept, say, 6 per cent. return on copper shares, and ask the same of gold. If such investors could bring themselves to abandon their attitude of self-deception for a moment they would no doubt admit that what they really want is share speculation, but in order to indulge themselves comfortably in such pastime, they seek to be told that it is "investment."

Taken broadly, the value of r''' is essentially the

* These formulæ are offered simply as illustrating one of the many factors making up the risk involved in any undertaking marketing a product varying greatly in price.

practical measure of speculation, even though no exact line be drawn.

Table I. gives what appears at first sight to be the risk-rate (r''') demanded by the public of four well-known Rand shares; the examples given have been taken at random. The amazing fact will be noted that buyers appear to be content to ask less of these mines than of standard railway shares, thus tending to confirm the cynical view that the public finds in mining only a convenient table upon which to gamble.

TABLE I.

Company.	Life in Years.	Rate of Dividend on Par Value.	Rate Paid on Market Value of Share.	Rate Necessary to Redeem Capital.	Rate compared with 3 Per Cent. Consols. + Higher. − Lower.
		Per Cent.	Per Cent.	Per Cent.	Per Cent.
A	30	40	7	2	+ 2
C	8	30	14	11	0
D	16	35	9	5	+ 1
E	12	17	9	7	− 1

Average risk-rate, half of 1 per cent.

Figures taken from the *Mining Magazine*, March, 1910.

While no doubt there is truth in this, the moment an engineer admits that he has no concern in providing a good return on capital, this essentially meaning r', r'', and r''', he becomes but a croupier in the game, as it were, and his position in society should be, and often is, so regarded.

Reverting to Table I., showing the Rand risk-rate as

apparently estimated by the public, one has to take several things into consideration before deciding what is demanded.

First, ignorance of the effect of "life" allowance in increasing the rate to be legitimately asked; second, the degree of risk run; third, the imperfection of the data upon which the investment was made; fourth, the disinclination of shareholders to reduce the value as set in a time of speculative hysteria when the major part invested; fifth, the hope that others more venturesome than themselves might yet take their holdings.

The very fact that prices paid for mining shares often, if not usually, imply the loss of part of the capital embarked, demonstrates the needs of the consulting engineer as a technical adviser, whose first duty should be to estimate, however approximately, the value of r'', r''', and r''''. Those who consider the latter in their daily calculations are too few in number, unknown to many, and have status only as special advisers to the larger and more reputable concerns. Their light often has a bushel kept carefully over it, lest the public, learning of their existence, should demand favourable reports from them on new ventures.

It may be contended that it is useless to employ a risk-rate when so many factors influencing it must remain indeterminate; as, for example, the various personalities effecting the enterprise.

On the contrary, because of the possibility of the adverse factors synchronising, as it were, it should be the more advisable to use it. The varying weights given to the reports of different engineers is expressed to a certain extent by the market values of shares in ventures

recommended by them; this is an unconscious employment of a factor of safety, but expressed in terms of present value.

Where engineers check one another's work, a rate of risk applicable to each other may on occasion be approximated; for instance, the results obtained from a reconnaissance inspection of a Chilean nitrate pampa indicated that by using the data from the careful examination of a few points as unity, the accuracy of an earlier report might be allowed 0·66, and on this basis the pampa was given 18,000,000 quintals, while the final and exhaustive examination by a third engineer gave 20,000,000.

Again, in the preliminary examination of the mine referred to by the Table II. and Fig. 1, the errors found in the methods of sampling alone gave rise to the prediction that others yet more serious would be found in the block calculations.

TABLE II.

Classes of Ore.	Part of Block Sampled.	Amount.	Net Value.	Ratio of Value to Purchase Price.	Ratio to Capital of Company.
		Tons.	£	Per Cent.	Per Cent.
Positive ore:	Patches.	175,000	182,000	17	14
Block A.	Top, imperfectly.	55,000	112,000	11	9
Block B.	Top and bottom.	77,000	200,000	19	16
Probable ore: Blocks D and E.	Half of top; virgin below.	318,000	560,000	53	61
	Totals,	625,000	1,054,000		

First payment made—£1,000,000.

This was borne out in the next work, which, though uncompleted, showed an error of £80,000, as was published subsequently.

The proportion of the so-called "positive" to "probable" ore, and of these to "possible" ore, when a purchase is recommended, is one that must greatly influence the value to be given to r''', and is often the best criterion of an engineer's qualities, moral and technical. The practical definition of each of the above

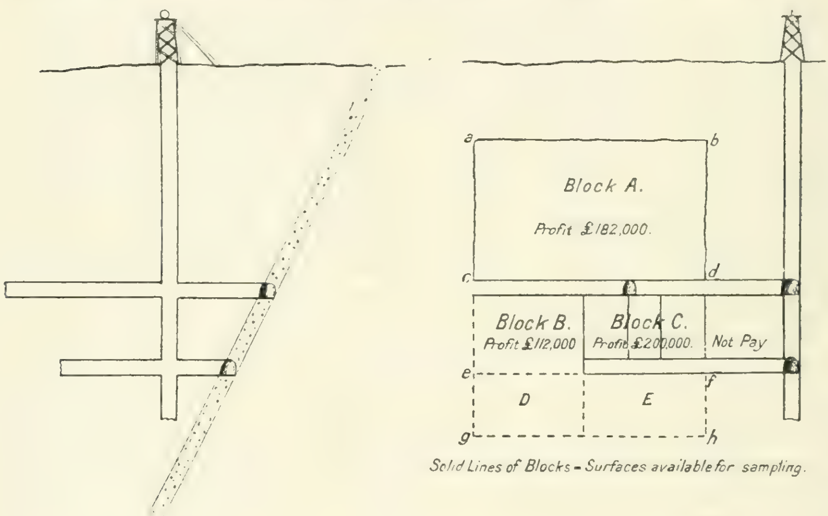


Fig. 1.

classes of ore varies, the interpretation of the local data, geological and economic, largely affects it, and unfortunately gives an opening for many plausibly to misuse terms and introduce a skilful ambiguity leading to a successful public exploitation.

It is a common practice among the more careful engineers to demand that the dividends from the "positive" or four-sided ore must be at least equal to the

Consols basis (*i.e.*, $r' - r''$), leaving the "probable" to represent r''', and, one might add, the "possible" ore, to give the zest so essential to embarking upon new undertakings.

The statement * that 7 per cent. is the minimum to be given to r''' in any class of mine investment is well worth careful consideration. In a general way one may say that the estimation of r''' is that portion of the reporting engineer's work which makes the greatest demand upon his knowledge of economic geology: for on the above basis, $r' - r''$ is covered by the ore actually blocked out. The sampling of the latter calls for little more than a knowledge of mining as practised or practicable locally, a good eye and a sound grasp of the principles of sampling and deductions therefrom, though the last would seem to be little enough understood, even by the fraternity.

From this aspect of the risk-rate (r'''), the probable occurrence of ore beyond the pick-point, whether it be the continuation of an ore-shoot in a typical vein, or the lateral extension of a mineralised part of the porphyry flow, though often calling for the most exhaustive study of the surrounding district, is the practical object of geological work. For instance, the idea that a fair allowance for shoot-extension would be a wedge having a base equal to the shoot-length on the lowest level, and terminating at a point distant one-half of the shoot-

* See "Principles of Mining," by H. C. Hoover. The above 7 per cent. for risk may be read as corresponding to the writer's $r''' = 4$ per cent.

length below,* is another estimate of r'''. The practical application of this allowance calls for the careful study of the type of deposit, as well as of adjoining mines, especially if they be deeper or near the same depth, and is admissible only if they show no cause to suspect a sudden diminution in the pay-ore.

Many will insist that mines may not be bought on a basis of the ore proved. This is often true, but in the

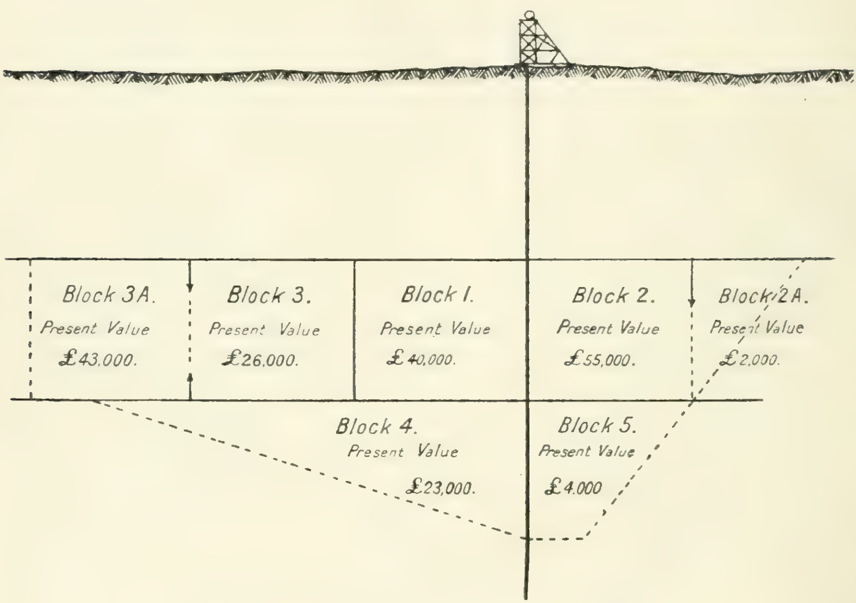


Fig. 1*a*.—Diagrammatic Sketch of Mine.

Solid lines = work done; dotted lines = estimated boundaries of blocks.

case of undertakings calling for a heavy outlay on equipment, terms approximating these can usually be made, especially if the engineer be possessed of tact, and has

* See "Principles of Mining," by H. C. Hoover.

the intelligent support of a financial group intent on making a good purchase.

Internal Factors of Risk-Rate.—As is indicated above, the risk-rate is composite, being made up of factors, one in effect is made a function of the time; another being a function of the specific data available. In the case of the ordinary banking risk, where a higher rate is asked of the individual than of another bank, the discount of a man's paper places a larger sum to the credit of the bank's reserve at the end of three years than if discounted for three months only. This is tantamount to saying that, apart from ordinary interest (say 3 per cent.) and capital redemption, the contributions made to meet possible loss shall be a function of the time, and independent of the risk appertaining to the individual himself. This seems defective, but it may be said that practical experience proves the number of events likely to vitiate the accuracy of a forecast, to be a function of the time; and that, instead of setting a constant as the special contribution to the reserve fund for each customer, beyond the bank rate, it has been found simpler and more effective to make the individual's contribution a function of the time. So with the unit value in blocks of ore carrying the same risk-rate, there are two risks; one external to the block, and representing the many risks incidental to war, pestilence, famine, and maladministration, hence a function of the time. The other might strictly be considered to be independent of time and purely a function of the data available for the block. For instance, if 2 per cent. (or 5 per cent return) were considered to be an adequate

external risk-rate, the aggregate contribution for the risk of one pound for ten years would be ·229 (after subtracting the amount at the Consol rate). If, however, the time ran to twenty years, and the same risk-rate of 2 per cent. were applied, the same reasoning would show that a contribution to the reserve fund of ·537 (allowing redemption and 3 per cent.) would be required. This is analogous to the banker's demand, and may seem defective, yet experience, and perhaps convenience, has sanctioned the custom of making this contribution a function of the time. In the case of blocks of ore having the same amount of data and same deferrence, but of a different profit per ton—hence a greater risk—special allowance might be made in accordance with the principles discussed in connection with Formula (18), but it would seem safer and certainly simpler to follow the banker's procedure. On rare occasions one sees a report having the net profit reduced in proportion to the sample surface thought necessary. For instance, where a winze and a level of a block are alone available for sampling, the tonnage and profit in block 4 of Fig. 1 (*a*) might be reduced by one half, and the present value determined on the basis of the external risk-rate only. This would obviate some of the manifest defects in the banking system, but would not allow for variation in profit per ton—*i.e.*, the attendant greater risks with ore having a value near the cost of production. On the whole, it would seem that, as in commercial life, it were better to make the contribution to the reserve fund, so to speak, greater in the cases of longer deferrence, always bearing in mind, of course, that the most one can hope is that the

under-valuation of certain blocks will be offset by the over-valuation of others. The really vital point is that a full presentation of both data and the methods of collecting same be submitted, together with the means of applying the facts to the estimate of present value; this, of course, is nothing more than a demand that the scientific habit of mind be observed, the essential of the latter being the elimination, as far as possible, of the personal factor.

The Redemption Factor.—The life factor, r'', or $\frac{r'}{R'' - 1}$,* the portion of the annual rate to be set aside to redeem capital in "n" years, naturally carries with it many practical considerations, one being the capacity of the reduction plant, which, of course, not only determines the life of any block, the tonnage of which is known, but largely influences the working costs, hence has signal bearing upon the blocks to be left, thus reacting on the life. Very much has been written on the subject of the most profitable magnitude of plant (or life), and while of great moment to those operating engineers, who may count upon more capital for the plant extension if needed, for practical purposes of block valuation, life is fixed by existing plant, or by the convenience of the financiers finding the capital. While a plant treating anything from 50 to 1,000 tons a day may be calculated upon in reporting practice, larger ones than this are usually matters of growth after further capital expenditure has been justified by reduction results already obtained, as well as by greater proved tonnages. Naturally, the

* Sinking Fund rate (see pp. 24 and 25).

promised magnitude of the deposit must largely influence the size of the plant, but usually the financial weight of those for whom the report is made is the dominant factor, though evidently sound field work will record data in such a way as to permit of calculation based on various sized plants. Few things are more noticeable than the practical disregard of the redemption factor, not only in the valuation of shares and blocks of ore, but by the operating engineer in secondary calculations appertaining to plant additions. It seems certain that were this feature to find expression in all estimates where relevant, a much clearer idea of the real cost of improvement would obtain than where the lesser capital outlays are charged to renewals. In fact, were it not for the tendency to surreptitiously charge off working expenses to capital outlay, such an account could well be introduced with a view to commitment of the manager to intelligent estimates, these involving not only a statement of the savings in costs expected through the outlay, but the presentation of the real gain after redeeming the latter in a few years.

The Deferrence Factor.—When surveying current valuation practice, few things are more remarkable than the studied avoidance of the deferrence factor (r''''), though the engineer in touch, even superficially, with finance has it thrust upon him when either cumulative shares or debentures are mentioned. The former of these two holdings essentially implies that though dividends be deferred at the outset or suspended later, the loss of simple interest must be made up by the dividends when these are resumed. In the case of debentures, the right

of foreclosure as soon as interest is defaulted involves the same principle coupled with that of capital return. Even apart from the elementary common sense so aptly applicable, the above fact that the principle is daily recognised on the share market, renders the ordinary engineer's neglect to apply it to the valuation of reserves the more inscrutable, and no doubt gives rise to the attitude of amused contempt with which intelligent financiers regard his pose of an all-round business capacity.

It is the rarest event to meet a report on a metal mine having the assets set out in such a manner as to allow for this simplest of economic principles, and the resulting magnitude of the error in valuation, if even suspected at other points in an examination, would not only condemn the engineer out of hand, but stamp his report as inept.

As seen by Formulas (4) and (15), or by Tables XXVI. to XLII., the deferrence principle tends to bring the value of certain blocks, in a property with a long life, to the vanishing point, which, however, may be offset by an increased rate of exhaustion should further capital for equipment be available.

While more pertinent to the economics of operation than to those of valuation, it may be mentioned in passing that the risk-rate and deferrence principle operate somewhat divergently in the case of expenditure on development. Here capital is laid out, upon which no return is made until the block is worked ; in other words, the risk-rate applicable to the block is, within the limits set by the standard dimensions, a function of the expenditure on

development. This is one of the many similar problems ever before operating engineers, who study the broader aspects of the work, but may not be touched now. As used here, *development* essentially implies exploratory work, or that undertaken to secure data as distinct from that properly charged to stoping such as many winzes, stop-drives, etc.

CHAPTER I.

BLOCK CALCULATIONS.

CALCULATIONS being numerical deductions from accumulated data, it may be urged that the nature and manner of collecting the latter should be first discussed.

As, however, calculations must rest on Premise III. and its corollaries, the more convenient sequence would seem to follow the latter, lest the underlying philosophy become obscured by what may be called pure technique.

The definitions of reserves being a body of ore or mineral concerning which sufficient physical data is available to warrant purchase, it follows that upon the interpretation of the data must be based the value. It is well, therefore, to have clearly in mind the significance of the word "physical." For example, by referring to Fig. 2, the suppositious case of a copper deposit capped by a worthless gozzan, it will be seen that when an engineer advises outlay, either for purchase or development before ore is actually reached, no physical data is available.

Those finding the capital may be said to support the engineer's or geologist's hope and prediction that payable ore in payable quantities will be met with at depth. It follows that if definite values may not be assigned a block of ore, it of necessity falls under the same category, regardless of its position in the mine; becomes one

of hope only. Expressed otherwise, while no such thing as certainty exists in any undertaking, the whole of business life is one of outlay *on estimates of probability*, hence the convenient expression "*possibility*" for the

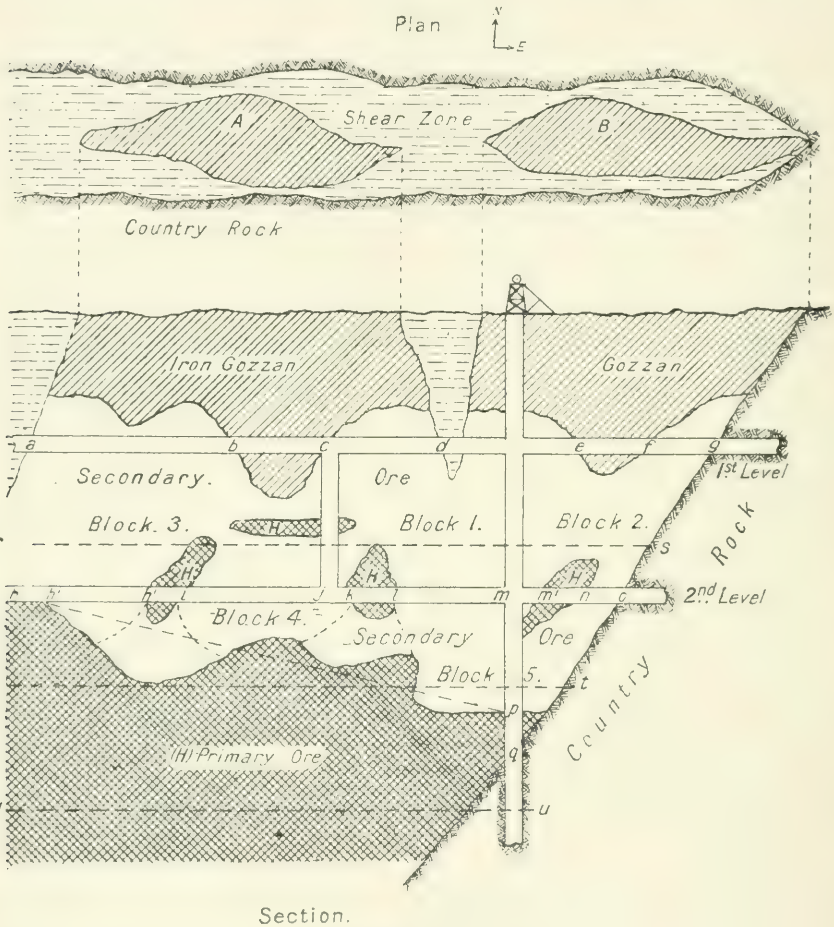


Fig. 2.

"*probability*," which is thought too remote to adventure upon, beyond that incidental to exploration.

Evidently the ore in the interior of a block, though

sampled on four sides, may be said to exist only in the imagination, but the fact that in the latter *continuity* only must be assumed, against *occurrence and continuity* in the former case, throws the block into the category of *probability, which may be paid for.* The philosophy bearing upon the amount of these payments has already been touched upon in the introduction.

Shoot Indications.—A feature of the greatest importance, though constantly overlooked in reporting practice, when dealing with undeveloped properties, is the area of the out-crop formerly mineralised, or that which otherwise indicates the magnitude of the ore bodies expected. For instance, the sum of the areas A and B of Fig. 2, other geological data being favourable, may be said to be a guide to the area of a horizontal slice through the ore bodies at $r-s$. While, as indicated by the sketch, this estimate may be erroneous, it is invaluable, not only as the guide above mentioned, but, when set out in a report, as an evidence and measure of sound field work.

A great advance would be made were it definitely recognised that the failure to furnish a plan and section sketch of the out-crop to scale; in other words, the neglect to supply the primary data to justify the outlay advised, was a mark of incompetence. In those cases where no brecciated zone or other surface indication is available, the only guide may be the magnitude of the bodies found in a similar horizon. For example, the wonderfully rich bonanzas of silver chlorides at Huautahaya in Chili occur at the juncture of small, usually barren veins, with a limestone-andesite contact. In this

case the only guide to the economic importance of new bodies sought for, would be the magnitude of those previously met with under similar geological conditions in neighbouring properties, yet this is pertinent data.

In general it is convenient to conceive of a slice one foot thick through the existing or hoped-for ore bodies, perpendicular to the major axis. If, instead of thinking of this in terms of its physical dimensions, but of its economic magnitude, a very effective unit by which to compare the possibilities of different properties is had. For instance, if the slice indicated by $r-s$ of Fig. 2 were 1,000 feet long by 15 feet thick, yielded £1 per ton profit, and the specific gravity constant were 15, the economic magnitude would be 1,000 sovereign-feet.

A concise statement, then, of the engineer's report advising outlay on development would be—" Based on the evidence of similar geological conditions known to me, and upon the area of the leached out-crop, I expect to meet the top of a shoot of about 1,000 sovereign-feet in magnitude, counting a sovereign a ton profit, at about 500 feet in depth. The expenditure for exploration work is estimated at £15,000, or £15 per unit of economic shoot, which, allowing a further £15,000 for plant outlay, works out at £30 a unit, thus comparing favourably with other mines ; see Table IV. attached."

The above is a case of the estimation of possibilities and presents, aside from extra plant allowances, no difference from that of the " possibilities " of the shoot below the deepest level in a mine, *not ranked as* " *reserves*."

If, however, the out-crops A and B of Fig. 2 be conceived to be adequately sampled, and to carry gold

values payable under the local conditions and plant equipment financially convenient to those for whom the report is made, the whole aspect of the case is changed, insomuch as physical data is in hand as distinct from the geological surmise regarding the copper contents below. In other words, the position, apart from provision for plant, is similar to the deepest full-shoot level in the typical mine—*i.e.*, where data from sampling one side of a block of ore is available.

The Standard Block.—When calculations refer to a block of ore having the whole vein-width exposed on four sides, one is immediately confronted, not only with the distance between the sample sections as determined by the homogeneity of the ore, but with the depth of enrichment perpendicular to the sampled surface. In other words, one can sample, as a rule, but four of six sides of a block, hence investigate but a shallow zone on the periphery, the depth of which may not be assumed greater than the distance between the sample-sections.

It may be said that the ratio of the cubic contents of this zone to that of the block is a measure of the uncertainty attaching to the valuation, if the above view hold good. From this it will be seen that to be fairly sure of a block, the dimensions may not be greater than the depth of impregnation, as indicated by the intervals between the sections, thus reducing the size of the blocks by an amount impossible in mining practice. For instance, if it were not allowable to take one section more than 20 feet apart, one would not be justified in spacing levels and winzes more than 40 feet distant one from the other.

The above view would not seem adequately to cover the ground if we study closely almost any carefully prepared essay-plan wherein the sections are, for example, 5 feet apart. Here it will be seen that there is a general tendency of the ore found in the shoot, which alone is now being dealt with, to maintain for several sample-sections a yield, let us say, barely profitable. The next number of sections may be rich, and so on, around the whole periphery of the block. If those parts carrying the same value be coloured, it will be seen that in many ore-shoots at least, there is a tendency of the leaner (or

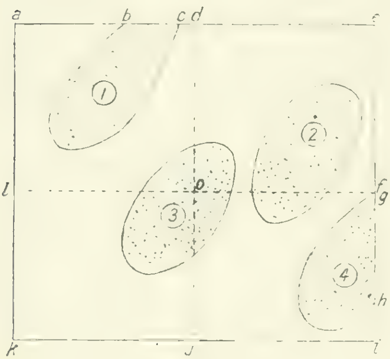


Fig. 3.

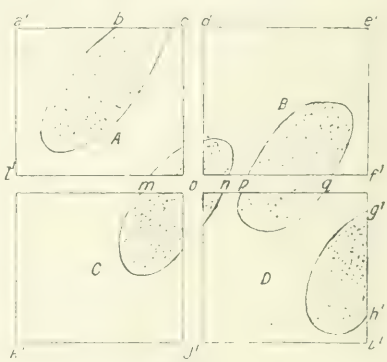


Fig. 4.

richer) portions to assume a form roughly indicated by Fig. 3, which aims to illustrate such a block of ground.*

After a little consideration of the principles governing the weighing of data, it will be seen that in order to give the minor areas indicated by (1), (2), and (4) of Fig. 3 their proper weight in the calculations, the number of sample sections taken in the distance *b-c* and *g-h* should bear the same ratio to the total number of periphery

* The shape and magnitude of these patches, as indicated by the assay-plan, is manifestly worth the most careful attention.

sections as the areas indicated by the dotted portions bear to the area of the whole block. As will be seen from Fig. 3, however, the distances between *b-c* and *g-h* form but a small proportion of the total periphery—that is, areas 2 and 3 are unsampled, and fail to affect the mean value of the block as determined by sampling the periphery *a-e-i-k-a*.

If we divide the block as is shown in Fig. 4, it will be seen at once that the minor blocks A, B, C, and D contain each of them a portion of the areas 1, 2, 3, and 4; hence a nearer approximation to the mean value of the whole block is obtained by sub-dividing it—*that is, the accuracy varies with the data available, or the risk varies inversely.*

Theoretically, of course, complete accuracy is never obtained until the whole block is carefully crushed and sampled, the essential principle involved being the same as that governing the quartering of a sample. Inasmuch, however, as this process is so expensive in mining work, it is necessary to make a compromise between the demands of theory and the same when economically weighed so to speak; expressed otherwise, the exigencies of mining largely determine the distance between levels and connections. Nevertheless, should a preliminary sampling show such variations or patches as are indicated by Fig. 3, it would be either sub-divided or an allowance made approximating it, as a theoretical equivalent. In other words, depending upon a preliminary sampling or upon a judgment based on previous experience with similar ore-shoots, consciously or unconsciously one has to decide upon the dimensions of

a standard block—that is, having a given ratio of periphery to cubic contents.

The writer's practice is to take sections at 30 feet intervals, sub-dividing these to as small as 5 feet, if marked variations either in thickness or value are noted in the first series. The great number of sections usually taken is due to an unconscious effort to minimise the tendency of accidentals to vitiate the mean value, as well as to define the habit of the rich and poor patches.

While the demand incidental to winning the ore largely affect the distance between levels, the general habit in vein mining is to space levels about 100 feet apart, even where this distance could profitably be increased, having reference to the mining cost alone. This spacing of levels is no doubt partially the outcome of generations of experience in the matter of variations in value within the ore-shoot, and in general it may be considered safer to space winzes further apart than the levels.

It will be apparent that the conditions illustrated by Figs. 2, 3, and 4 are fairly common to tabular deposits, though the shape and size of the lean patches will vary greatly as affected by secondary enrichment and impoverishment, by cross-fissuring, contacts, and a host of other geological factors beyond the scope of this work.

Fig. 2 illustrates a not unusual occurrence of ore bodies in the typical copper mine where one ordinarily meets greater changes in the same distance than in gold or even silver deposits. In such a case the allowances made by different engineers for each block would doubtless vary, but where full plans accompany a report,

coupled with detail, as is set out in the tables and elsewhere, adjustments can easily be made.

Block 4 of Fig. 2 especially illustrates a case of where geological data, as indicated in the second level and at the points p-q in the shaft, would tend to prevent any such allowance for shoot-extension as set out in Formula (22). On the other hand, the occurrence of the secondary ore from h' to m and from m to p, would seem to justify a tonnage allowance, shown by the dotted line h'-p, weighted in accordance with the principles to be set out. It will, of course, be evident that, under certain circumstances, largely depending upon the information furnished by the assay plan, greater weight might be given to the same length of sampled periphery on a level than in a winze. On the other hand, were the richer or leaner portion to take the form of sills, like that of the body marked by the upper H in block 3 of Fig. 2, equal or greater weight might be given the winzes.

The point urged is not the fixation of rules like this, local conditions greatly modifying these, but the necessity of setting out the fullest particulars. which must include not only the field data, but each step in the calculations and deductions, in a manner indicated by the discussion of the following premises.

Those familiar with the Transvaal will recognise at once, by comparing the shoot conditions found on the Black Reef and Pilgrims with those of the Central Rand, the significance of the different dimensions allowed the standard block. In parts of the Rand one might feel safe in allowing 500 by 500 feet, and feel doubtful of 50 by 100 feet in another geological district.

If the necessity for, and significance of, the dimensions of the standard block be seen, it becomes possible to repeat with greater emphasis the contention that—

*The risk-rate (r''') for each block varies inversely with the number of sample sections taken.**

For instance, were the dimensions of Block A in Fig. 4 set as a standard when sampled around its whole periphery a'-c'-o'-l' at, say, 10 feet intervals, then the r''' selected would be a corresponding standard value. If, now, Block B were valued on the basis of sampling, only the surfaces d'-e' and e'-f', our data would be but one-half that demanded as standard, hence the r''' would be doubled. Again, if only k-j-i were sampled, and the dimensions of A be still taken as the standard, then the ratio of the data available to that demanded would be as the distance k-i is to the whole periphery a-e-i-k plus l-f plus d-j, and the risk-rate (r''') allowable would be six times that given to the standard block.

From the foregoing it will appear that, while there is room for considerable divergence of opinion regarding the dimensions of the standard block applicable to

* It will be noted that this theorem may be written algebraically, $r''' = \frac{k}{x^n}$, where $k = 1$, $n = 1$, and $x =$ the number of sample sections. Others, of course, may assign different values to k and n. The above theorem might be written $r''' = \frac{k'\,k''}{x}$ for each block, where $k' =$ the risk-rate for the standard block, k'' the number of sample sections required, and x the number of sections taken.

each ore-shoot, a carefully prepared assay-plan will often shed a most illuminating light on this as on other points.

The value to be accorded the risk-rate (r''') for the standard block again calls for consideration, and though a multitude of influences affect it theoretically, a low value can usually be given with propriety, especially when dealing with gold properties, though in the case of copper and tin the law indicated by Formula (17) might be considered.

On the assumption that the dimensions of the standard block have been carefully determined, in the case of a gold property it would appear that a risk-rate of 3 per cent. should be sufficient, though this must cover the danger of losses through war, pestilence, famine, theft, strikes, and adverse legislation, all exterior to the mine itself.

In the case of a property similarly situated, the dividends from which are principally dependent upon the sale of silver, a risk-rate of 5 per cent. would appear a more reasonable allowance, particularly if the profit per ton were small.

Shoot Extension.—As the ore-shoot extension allowable below the deepest level is the source of the greatest doubt regarding the value of a mine, hence the root of most parasitic financial growths, definite criteria of sound valuation are impossible without determination of such allowance. A formula expressing the above fundamental principle in simple form applicable to extension of the typical ore-shoot may be written—

Formula (19). $r''' = \mathrm{P}\left[\frac{(l + y)\, l' + k\, l\, y}{l'}\right].$

Formula (20). $\quad = \mathrm{P}\frac{k\, l\, y}{l'} + \mathrm{P}\,(l + y) = \mathrm{P} + y\left(\mathrm{P} + \frac{\mathrm{P}\, k\, l}{l'}\right),$

where P = the standard risk-rate ;

y = the number of zones or levels below the lowest one proved ;

l' = the length of shoot as shown by the lowest level ;

k = the number of winzes there should be between two levels in order to make them into standard blocks ;

l = the distance between levels on the dip.

Referring to Fig. 3, and assuming that $k - i$ were the shoot length on the lowest level, and P were taken at 5 per cent., $l' = 200$ feet, and $l = 100$ feet.

$$r''' = 0{\cdot}05\ \frac{(1 + 2)\ 200 + (3)\ (100)\ (2)}{200} = 30 \text{ per cent.}$$

Reverting to the case last cited, where it was found necessary to allow a risk-rate of 30 per cent., in order to weight the uncertainty attaching to the ground below the last level sampled, it will be seen at once that if our dividends were only 30 per cent., the principle of probabilities as incorporated in the above theorem would limit the shoot-extension to two levels.

From the foregoing it follows that the old uncertain nomenclature of "probable ore" is discarded, but that probability is defined in terms of annual interest—that is, it has now an exact financial equivalent. Among other points of practical significance attaching to the same definition of probability is the fact that, *under*

certain circumstances, the ore below the lowest working must be regarded as a reserve, when properly weighted, the same as any other block, but of even greater moment is the fixation of the shoot-extension that may be allowed on a given estimate of dividends to come from that zone.

In order that the above principles be given adequate expression, it is necessary to give greater attention to the effect of deferrence with reference to determining the present value of each block, as set out in Formula (15), which is the bases of Tables XXVI. and XLII.

Returning to the shoot-extension allowable below the deepest level, and neglecting for the time being the effect of r'''', the basic Formula (1) may be written—

Formula (21). $\mathrm{D} - r' = r'' + r'''.$

Formula (22). $\mathrm{D} - r' = \dfrac{r'}{\mathrm{R}^n - 1} + \dfrac{\mathrm{P}}{l'}\left[(l+y)\,l' + k\,l\,y\right].$

where D = the average annual dividend in percentage of the capital ;

r' = the Consol rate, or 3 per cent. ;

r'' = the rate necessary to redeem unit capital in n years ;

$= \dfrac{r'}{\mathrm{R}^n - l}$;

R = $(1 + r')$;

n = the life of the mine, n years ;

r''' = the risk-rate ;

r'''' = the rate necessary to make up the loss of interest during the deferred period.

This, unlike Formula (19), takes into consideration

the part of the annual dividend that must be set aside to redeem unit capital in n years, as well as the risk-rate.

Evidently D, the annual dividend, must be on a certain capital, which implies a known crushing plant that in turn determines the life that may be allowed to each zone or distance between levels l, hence n of the above equation can be expressed in terms of y, the number of zones, and be written $k'y$, where k is the number of years' life allowed to each level.

In the case of going mines where part of the capital has been repaid in dividends from the upper levels in n' years, n of Formula (22) may be written—

Formula (23). $$\mathrm{D} - r' = \frac{r'}{\mathrm{R}^{n' + k'y} - 1} + \left(\frac{\mathrm{P}\,k\,l\,y}{l'} + \mathrm{P}(l + y)\right).$$

As y also appears in the exponential form, solutions are obtained by the trial and error method, but it will be seen at a glance that the above simple formula, based on the ordinary insurance concept inverted, *limits the depth to which sound practice may allow shoot-extension, yet places this ground in the category of a reserve.* At the same time, it takes into consideration such factors as shoot-length and thickness, the profit per ton, and the crushing capacity of the plant, the number of levels already exhausted and the "patchiness" of the ground in the ore-shoot.

The above may be summed up in the statement that allowance for shoot-extension into unknown ground should be a *function* of the *magnitude of the economic shoot*, instead of purely a function of the physical dimension of the shoot, as is loosely held by present practice.

This will be seen to be in harmony with the ideas expressed in Formula (17), though very brief thought should show the fallacy of the older concept.

As will be seen, Formula (23) demands a greater allowance for shoot-extension in the case of rich than of poor ore-shoots, which also is in accordance with probabilities as just touched on.

It may be well to call the casual reader's attention to the fact that the above formula is *not a certain prediction of the shoot-extension to be encountered, but is a simple statement of what is thought to constitute sounder practice in estimation of probabilities*; as mentioned before, one may successfully wager ten to one that the ace of hearts will be the first card in a pack, but another may well question the wisdom of the hazard as of the man taking such chances, even should he be successful.

The Economic Shoot.—In the ordinary course of mining operations, the successive economic stages may be roughly summarised as follows, though the varying degrees of development in different parts of a developed mine may involve problems peculiar to an earlier stage. These may be said to be—

(1) The period of purely geological surmise;

(2) The period of definite quantities and values, as well as of surmise.

Finance, at least such phases of it as legitimately concern the reporting engineer, calls for a careful consideration of the above stages, and of the first—that of geological surmise—one may write without much fear

of contradiction. "Unless profitable ore be found on the surface, sound valuation does not permit of cash expenditure, save on exploratory work." Manifestly this implies the belief that the ore bodies will repay the outlay necessary not only to exploration, but to development and plant, wherewith to realise the values contained; in other words, even the stage of geological surmise implies a postulate of dimension and value.

The first stage is more especially noticeable when dealing with copper outcrops, where profitable ore is rare at the surface.

On the other hand, in the case of, say, a gold prospect stripped along the outcrop and sampled, one may be justified in paying the vendors in cash, the amount of which would depend upon the profit per ton, length and thickness of shoot, and capacity of plant, the value of which must be redeemed, beside other factors.

It follows, then, that payment for property is not a question of development, but of data available, and it is conceivable that a well-sampled outcrop with favourable geological conditions may be given the same value as the deepest level in a mine.

If it be admitted that expenditure on the strength of geological surmise unsupported by assay values, other than that on exploration, be unsound, then a purchase involving payment for ore below that allowable as determined by the assay-values, is also unsound.

Stripped of technicality, one may say that where outlay is made independent of values, the investors are "backing" the geologist's surmise, and while they might cheerfully support a specialist of the first rank, it is

doubtful if sound practice would permit payment to vendors before ore was found. Furthermore, the specialist would have at least some idea of the economic magnitude of the ore-body expected, whether a "saddle" at Bendigo, or a zone of secondary enrichment below a brecciated gozzan.

If the foregoing be accepted, then the expenditure on exploration per unit of ore-body expected should be a measure in current practice. As it would seem to be impracticable to predict the depth allowable unsampled or "surmised" ore, the unit used will be the sovereigns of profit in a horizontal slice through the ore-shoot, one foot thick. This is a basic feature common to both stages, and will be called the "economic shoot"; as it serves as an index and means of classifying the possibilities of each, and should be clear from the following figure, which reproduces a common type of undeveloped prospect, having in general an appearance like that shown in (*a*) of Fig. 5; (*b*) gives a longitudinal section of the same when developed to, say, the company stage.

It would seem that the most casual speculative thought would go a step beyond the idea conveyed by the ordinary word "shoot," insomuch as the boundaries of the latter are necessarily economic.

Surely the most obdurate must admit that a term, which in its very essence is economic, yet which conveys no definite meaning, suggests either stupidity or intention to mislead. Surely, when a shoot may be, say, 1,500 feet long, and yet be either 6 inches or 60 feet thick, when it may yield 6 pence or 6 sovereigns per ton in profit,

the use of mere shoot-length as a measure of magnitude is ineffective, to say the least.

This estimated dimension of the economic shoot applies, not only to the prospect, but to the developed

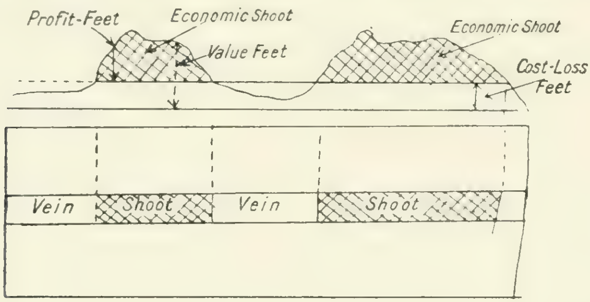


Fig. 5*a*.—Surface of Prospect—Plan (Syndicate Stage).

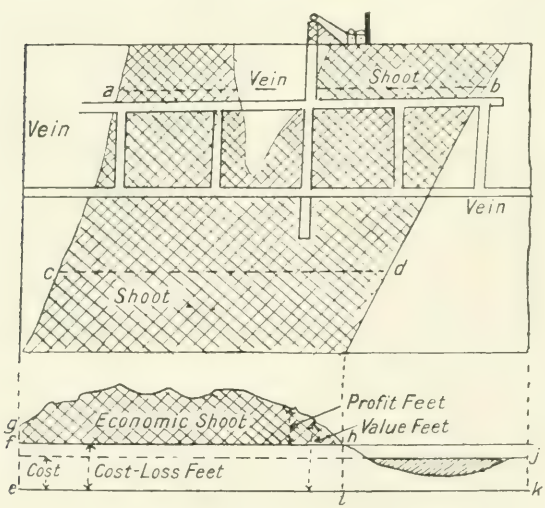


Fig. 5*b*.—(Company Stage) Longitudinal Section.
Area $efghie$ = Gross value shoot.
Area $fghf$ = Economic shoot.

mine; it is often as applicable to iron as to gold; to china clay as to copper; to a bed of banket as to the

channel of an ancient river; whether the major axis be horizontal, vertical, or inclined.

The one point of difference to be made between prospects and mines, between the syndicate and company stages, is that the latter usually calls for definite assets or "blocked ore," and, if well financed at the time of purchase, has the economic shoot with its possibilities thrown in.

Classification of Prospects.—Prospects may be conveniently divided into two types, depending as in all such classification upon the validity of the data offered. These are—

(*a*) The geological surmise type, as, for instance, petroleum prospects in new fields;

(*b*) Proved shoot type; with ore to be sampled on one side only, as, for instance, an ancient working in Rhodesia accessible to sampling; or a well-exposed outcrop.

In the case of (*a*) it may be stated that only upon rare occasions could it be considered sound practice to lay out capital save for development work, while, as before mentioned in dealing with (*b*), if local geological data were favourable a payment to vendors might be made in accordance with the principles already enunciated.

From the foregoing it will appear that finance during both the syndicate and company stages is vitally concerned with the magnitude of the economic shoot, while the ratio of that portion of the paid-up capital of the company not represented by the present value of the

proved ore, to the units in the economic shoot, furnishes a means of comparing valuations.

It will have been noted that "probability" as a term defining ore-bodies has been eliminated, it having been incorporated in the risk-rate principle. There remains, however, "possibility," which, as before mentioned, refers to the economic shoot below the ore allowed for. For example, when considering two mines having different economic-shoots, the possibilities of securing the always hoped-for bonanza in the ground beyond that for which payment may be made, might be said to be greater with the bigger economic shoot, as a single foot in depth would yield a greater profit in the case of the greater shoot. For instance, in Fig. 5*b*, though we may pay for the ore to the point *c d*, the ore beyond is "possible" only. In other words, we pay for probability, but hope for possibility, and one measure of ability is the price paid for a unit of this comforting sensation, as discussed later. (See Table IV.)

The foregoing may be said to bear particularly upon what is permissible, and while ideas will vary somewhat both as to the risk-rate applicable and as to the dimensions of the standard block, once these are determined, there would seem to be small room for great variations of opinion as to what constitutes sound valuation.

Economics of Purchase.—Evidently, the measure of the reporting engineer *per se* is his ability to comprehensively express technical facts and postulates in economic units, in order that outlay should be made in such a manner as to incur the least risk, or to invest the least capital incidental to securing a given profit.

Where the present total value at a low risk-rate is paid to the vendors of a mine, it may be safely said that either the mine was over-valued by the engineer, or the financiers neglected the very elements of their work, which are intelligent negotiation after valuation. It must be confessed that even the degree of self-appreciation common amongst engineers cannot obscure the fact that a little intelligent handling of the personal factors governing the vending interests, thus obtaining a deferrence of payments, will often secure benefits that make ordinary technical economies seem paltry; this is too often entirely overlooked. As before mentioned, an opinion as to the value of shares, large blocks of which are often offered for underwriting, is a constant demand, and while the basement complex of personality which affects the market and division of profits from such sources, concern the financier only, the economics of the enterprise, as well as the training, experience, and good name of the engineers upon whose recommendation flotations are made, concern the consulting or reporting engineer. From the foregoing it follows that, not only the relation of the proved ore-bodies to the purchase price plus equipment outlay must be considered, but also the payment made for the possibility of the economic shoot, though the latter can only be ascertained by combining a study of company construction and technical reports.

Economics of Finance.—Table III. gives a list of five well-known companies, with the approximate magnitude of their economic shoot expressed in sovereign-feet.

TABLE III.—THE ECONOMIC SHOOT.

Mine.	Metal.	Length of Shoot.	Average Width.	Gross Value per Ton.	Mean Sectional Value.	Combination Factor (Loss and Cost).	Mean Sectional Profit-feet.	Sovereign Shoot-feet.
		Feet.	Feet.	$	Feet.	$		
Pilares (Mexico),	Copper	1,700	600	8·10	4,860	6·1	1,200	27,000
Mt. Morgan (Queensland),	Gold-Copper	800	500	13·70	6,850	8·87	2,330	25,000
Santa Gertrudis (Mexico),	Silver	1,500	15·5	23·20	360	10·8	193	4,000
Mother Lode (B.C.),	Gold	1,200	150	6·70	1,012	5·7	150	2,500
Camp Bird (Colorado)	Gold	1,700	4	31·50	126	15·4	64	1,500

Table IV. may serve to emphasise the significance of the economic shoot as expressed in terms of company-capital and market-value. The present value of the reserves—that is, the deferrence factor—is neglected for simplicity's sake, and round numbers only are given.

To those also interested in the economics of finance, Table IV. should be of interest, though through lack of the latest facts the figures given must not be read as being exact, the idea being illustration from current practice, and not information concerning the mines mentioned. In some cases costs and losses have not been given in the official statements, and these have been approximated, as for example, in the case of the Prestea.

Columns 3, 4, and 5 are the proved shoot-lengths, mean thicknesses, and profit per ton, as given in published plans or statements only—that is, they are not

TABLE IV.—Valuation and Finance of Possibilities.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Name of Mine.	Metal.	Aggregate Length Proved Ore-shoot.	Thickness in Ore-shoot.	Estimate Profit per Ton Lowest Level.	Approx. Number of Sovereign Shoot-feet.	Approx. Present Profit in Reserves above Lowest Level.	Issued Capital of Company.	Capitalisation per Shoot-foot.*	Capital, less Reserves.	Engineers' Valuation of Possibilities. Rate per Shoot-foot.†	Market Selling Price of Co. (approx.)	Market Selling Price less Value Reserves (present approx.)	Finance of Possibilities. Price paid per Sovereign-shoot-foot or Depth to Recover Capital.
		Ft.	Ft.	£		£	£	£	£	£	£		
(*a*) Abosso (West Africa),	Gold	3,200	4·0	0·50	500	125,000	400,000	800	275,000	800§	550,000	425,000	850
(*b*) Prestea A. (West Africa),	Gold	4,000	7·0	0·85	2,000	500,000	850,000	425	350,000	425§	1,275,000	775,000	390
(*c*) Gt. Boulder Proprietary (W. Australia),	Gold	1,500	7·5	1·75	1,500	1,287,000	175,000	117	. .	117§	1,750,000	463,000	310
(*d*) Santa Gertrudis (Mexico),	Silver	1,500	15·5	2·50	4,000	500,000‡	1,275,000	(300)*	775,000	194	1,275,000	775,000	194
(*e*) Redjang Lebong (Sumatra),	Gold	900	40·0	1·90	5,500	110,000‡	200,000	37	90,000	17	1,600,000	1,030,000	187
						570,000							
(*f*) Alaska Mexican (United States),	Gold	1,000		0·36	8,500	375,000	180,000	21	. .	21§	540,000	165,000	20
			300·0										
(*g*) Alaska United (United States),	Gold	800	300·0	0·34	9,000	310,000	180,000	20	. .	20§	540,000	230,000	26
		350	300·0	0·30									
(*h*) St. John del Rey (Brazil),	Gold	800	17·0	·64	700	724,000	656,000	(940)	. .	. .	525,000	. .	0

* Column 9 is significant only if there be no reserves at the time of flotation.
† Column 11 is also only of interest at the time of flotation, but is then an accurate measure of both engineer and financier.
‡ This applies to time of flotation. § In default of relevant data this assumes no reserves at time of flotation.

private estimates. Column 6 is the keynote of the table, and, as before explained, is the measure of the ordinary mine, apart from the reserves, being the profit in sovereigns to be expected from each foot in depth as the proved ore-shoots are exhausted. It is held that this is a far more comprehensive measure of the probable future magnitude than the ore blocked out, as a small shoot may have been worked systematically and show large reserves, while a large one of equal profit per ton may have no reserves above the lowest level. In this figure of economic shoot are condensed the work and innumerable technical calculations incidental to sampling, assaying, losses, costs of transport, taxes, duties, refining charges, and a host of others, the enumeration of which obscures, often designedly, the real issue, which is the estimated future magnitude of the mine. Evidently costs and losses bear a relation to capital outlay on plant, hence such estimates are also incorporated in the economic shoot. The last word is used advisedly, for the boundaries of any shoot are essentially variables as determined by costs and losses, hence are economic, while the ordinary idea of shoot magnitude as expressed by area (length and thickness) does not express the profit per ton; it is therefore entirely misleading.

As the shoot bears on the future of the mine, and not on the proved reserves, the lowest full-shoot level is where one looks for data, and while the economic shoot at one level will vary from point to point, it remains an approximate measure of what one may expect in depth, assuming no vitiating geological features like the danger zone so common at about the same level in some silver

camps where zinc ores begin to appear. Occasionally, of course, an expanding or contracting rake of shoot, as indicated by assay-plans, might be considered, but usually this would be considered when dealing with reserves lying below the deepest level.

Column 7, giving the net value of the reserves, calls for little comment beyond suggesting that the validity of the estimates depends so much upon the size of the blocks and methods of sampling and appraisal, that one must take those of Table IV. as they stand. In reality, their actual value is inadequately expressed by the above figures, inasmuch as only their *present values* should be considered, *by allowing interest at a rate commensurate with the risk incidental to each until they may be realised upon.* For the purpose, however, of comparing the "possibilities," the above figures will answer.

According to the formulæ set out, the reserves in certain cases should include ore below the deepest level to a certain point to be determined by simple calculation. As, however, only those familiar with the local geology may say if such an estimate be permissible, those reserves lying above the lowest level are alone considered in the table.

Column 8 is of great importance as expressing definitely the valuation (when shares are issued at par) of the mine as estimated by the financiers, and to which, it is contended, the engineer is sponsor, more particularly with regard to the first flotation. This figure, coupled with the kind of shares issued (deferred, debentures, ordinary, etc.) is not only a measure of the engineer, moral and technical, but frequently a sure guide, if full

detail be published, to the business acumen, or the reverse, of the financiers responsible.

Column 9, or the ratio of the capital of the company to the number of sovereigns of profit to be expected from exhausting the shoots to a depth of one foot, may be said to be the payment made per unit of possibility, and is essentially the measure of business ability. To some it will be more convenient to regard it as the number of feet to which the shoots must be exhausted in order to repay the capital of the company (neglecting interest). This is another way of saying that those responsible for the flotation predict that the mine will not be exhausted before a certain depth is reached, for no one can conceive of risking money without believing that he will at least have his capital returned. *In other words, then, capitalisation beyond that represented by the present value of the reserves is equivalent to an estimate of shoot-extension in depth, whether this be on the engineer's advice or not.* In comparing new flotations, then, if no ore may be classed as a reserve, this ratio (Column 9) is the key to the economic position.

Column 10, or the issued capital (at par) less the reserves, is of particular interest only at the time of flotation, for the capital less the net value of the reserves must represent the amount paid for the estimated possibilities of the mine.

Column 11, the rate paid per shoot-foot, or the depth to which the shoots must be exhausted in order to repay the nominal issued capital less reserves, is also significant only at the time of flotation. For example, those responsible for the Santa Gertrudis flotation practically state

that independent of dividends the mine warrants a payment for shoot-extension to a depth of not less than 194 feet, while the financing of the Redjang Lebong called for 17 feet only. One would read it that the engineers and financiers of the Redjang Lebong must have looked to dividends, and an appreciation of their shares through development of the mine, rather than ask subscribers to gamble that the unproved ore would extend to depth.

Columns 12, 13, and 14, or the market selling-price of the mine, is affected by too many factors other than dividends, to bear upon those economics of finance with which we are dealing. On the other hand, column 14 is extremely interesting as illustrating what may be called the *finesse* of finance as distinguished from economics. This subject, however, calls for a subtlety of mind possessed but rarely by engineers.

The last four cases in Table IV. are of great interest, theoretically, and encouraging to those who follow mining for mining's sake, or who are concerned with the economics of finance.

It may be said that—

(*r*) Cash expended on an unproved unsampled prospect other than that incidental to exploration is unsound ; and

(*s*) The cost of exploration divided by the number of economic shoot-feet expected is a measure of outlay on possibilities ;

(*t*) Ore in a developed mine lying below that which the engineer is willing to class as a reserve,* and pay for, is "possible" only, hence is of the same nature as the unsampled prospect. Therefore, outlay other than for exploration is also unsound;

(*u*) The issued capital of a company (at par) less the value of the reserves divided by the number of units in the economic shoot, is the outlay made per unit of possibility, and is a measure of the finance valuation of possibilities.

The last is equivalent to the number of feet in depth necessary to exhaust the mine in order to repay the capital, hence a very fair idea, in default of modifying geological data, of the relative promise of capitalised mines may be had by applying the above (*u*); that (*r*), (*s*), (*t*) are not academic postulates finding no expression in current practice is shown by reference to column 11 of Table I., where it is seen that the capitalisation of the above-mentioned three great properties has not allowed for more than 20 feet of shoot-extension.

Those responsible for the last three flotations mentioned in Table IV. being known as exceptionally able and honourable engineers, the shoot-extension allowed may be fairly taken as what is thought to constitute sound practice, while the approximate agreement between them is of interest; nor should the fact be overlooked

* The above table classes as reserves only the ore lying above the deepest level. The writer holds that in many of the mines cited an allowance should be made for ore below, as already discussed.

that these extensions, of less than 20 feet, also include the promotional profits taken by the financiers concerned.

Referring to Fig. 1 and Table V., it will be seen, out of a capital of £1,275,000, only £346,000 was represented by the present value of the reserves above the lowest level. While nothing was mentioned touching the dimensions of what was thought a standard block, the fact that 174 sample sections were considered necessary to the valuation of the smallest block in the shoot, necessarily fixed the ratio of the number of peripheral sample sections to cubic contents for the others. This is a very important point to bear in mind in dealing with all reports where assay plans are published.

The upper reserves being taken at £346,000, leaves £929,000 as the estimated present value of the dividends to come from the ground below the deepest level, insomuch as not less than par was actually paid by subscribers. The economic shoot working out at 4,000 sovereign feet, the dividends from this ground should, allowing 1 level of 100 feet as worked per year, amount to £400,000 annually, or 43 per cent. on £929,000, which would begin three years later if, as would normally obtain, the upper blocks were worked first. The problem becomes: to what depth does a payment of £929,000 postulate the shoot-extension, assuming that the ground will yield dividends of £400,000 per annum, exhausting one level of 100 feet on the dip each year?

The above figures show that 230 feet must be allowed to recover the capital, but as no one adventures to secure a return of capital only, the real inwardness of

business concepts applied to mining risks remains to be considered.

The practical recognition of the deferrence factor implies that the dividend equation must be written—

Formula (24). $D = r' + r'' + r''' + r''''$, or

Formula (25). $D - r' = r'' + r''' + r''''$, which, by substituting the value of r'''' from Formula (6), becomes—

$$D - r' = r'' + r''' + (S^d - 1)\, r'' + (S^d - 1)\, s.$$

Formula (26). $D = (1 + r' + r''')^d\, (r' + r'' + r''')$.

In order to express the above payment in terms of shoot-extension, one may take a uniform risk-rate for the blocks of 10 per cent., manifestly too low, but sufficient for illustration purposes.

Substituting in Formula (26), allowing k' the value of 1—*i.e.*, one year's life to a level of 100 feet on the dip, and y the number of levels below, by trial and error method using Table XXIII. for the value of r'', it is seen that nearly 500 feet must be sunk below the then deepest level before the property will return the capital and 10 per cent. for risk. If one allow only 5 per cent. for risk, the shoot-extension must continue to a depth of nearly 400 feet.

The same solution and result may, of course, be had without calculation by referring to Table XXXIII., where it will be seen that, allowing 10 per cent. for risk, and deferring two years, the present value of an annuity, one pound is £2·46—*i.e.*, $n = 5$ years. This will be clear to those who have followed the elementary philosophy

TABLE V.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Block.	Ore.	Life.	Delay	Net Profit.	Annual Profit, Pounds Sterling.	Number of Sample Sections Taken.	Number of Sections Required.	Ratio, Cols. 6 to 7.	Risk-Rate (r''').	Factor from Tables.	Present Value, Basis A.	Present Value on Basis of Capital, £1,275,000.	Paid for Possibilities.	Remarks.
	Tons.	Years	Years											
A, .	175,000	1	1	£182,000	£182,000	135	750	0·18	·25	0·61035	£111,000	£1,275,000	..	..
B, .	55,000	1	2	112,000	112,000	96	174	0·52	10	0·69305	77,000	..	..	..
C, .	77,000			200,000	200,000	174	standard	1·00	5	0·79383	158,000	..	..	..
D, .														
E, .														
TOTAL,	..	..	..	494,000	..	..	..	..	..	..	346,000	..	£929,000	

underlying the development of $C=\frac{D}{r}$, and the division of r into its constituent parts to allow for ordinary remuneration, risk, redemption, and deferrence.

For instance, in the last case D = 1, and r = ·43, hence $\frac{1}{\cdot 43}=\frac{1}{r'+r''+r'''+r''''}=2\cdot 32$, therefore the nearest present value, when the deferrence is three years, gives nearly five as n, or the number of levels in question.

TABLE V*a*.

Block.	Net Tons.	Years of Life.	Years of Deferment.	Net Profit.	Number of Sample-sections	Risk-Rate.	Factor from Table 26.	Present Value.
						%		
1	50,000	1	2	£50,000	100	5	0·79383	£40,000
2	60,000	1	3	80,000	75	7	0·68301	55,000
2A	10,000	1	4	5,000	25	20	0·35520	2,000
3	40,000		4	40,000	80	6	0·64993	26,000
3A	60,000	1	5	90,000	50	10	0·48032	43,000
4	70,000	1	7	50,000	75	7	0·46651	23,000
5	20,000		6	10,000	50	10	0·42506	4,000
Total	..	..	..	325,000	..	..	.	193,000

Narrow Veins.—The first thing presenting itself, when the veins to be mined are so narrow as to necessitate stopes wider than themselves, is the possibility of leaving portions of the block unstoped; in other words, a process of sorting which does not imply first breaking the ground, a consideration almost universally overlooked in the valuation of reserves, pointing either to indifference or to a lack of underground operating experience on the part of reporting engineers. Yet before the extent to which

this is possible may be determined, the amount of ore which may be hand-sorted in stopes and on surface must also be approximated as well as the costs of the different operations. As will be dealt with later, under Premises IV. and V., the flexibility, or facility, with which the stopes may be contracted in strike and dip, is also a factor, the value of which is a matter of practical mining knowledge.

The question of costs will not be dealt with, it being assumed that the reporting engineer has had enough experience in various countries and conditions to approximate these. In general, however, particularly in parts remote from railway transport, there may be said to be a tendency to underestimate costs, the usual habit being to base them on those of a similar property operating elsewhere. Above all, in estimates of construction this transportation factor is given too little weight, not so much through the failure to approximate the cost per ton for carriage, as through overlooking the heavy losses due to idle staff and men, where plant fails to arrive as arranged for.

With reference to the possibilities of hand-sorting in stopes, underground practice will determine this to a large extent, such as the necessity for fills, size of waste broken, value and amount of fines, etc. When dealing with narrow veins, however, because of the superior supervision and mechanical facilities available on the surface, the usual custom is to sort there, and this will be dealt with, though the principle involved is identical both for underground and surface sorting as, in fact, for subsequent metallurgical operations. The first essential

is to determine if the advantage obtained by sorting offsets the drawback of multiplying operations.

Let B = the assumed (+ or −) profit per ton through sorting;

P = the percentage sorted out. (Of the ore sent to the surface.)

Q = the tons of quartz sent to the surface;

W = the tons of country rock or waste sent to the surface;

S = the cost per ton for sorting. (Over the tonnage sent to surface.)

L' = the total loss in the part sorted out divided by tonnage sent to surface;

C'' = the total cost of reduction operations (after sorting) divided by tonnage sent to the surface;

L'' = the value in the final rejecta, divided by the tonnage sent to the surface.*

This may be written—

Formula (27). $B(Q+W)+S(Q+W)+L'(Q+W)+C''(1-P)(Q+W)+L''(1-P)(Q+W)=(C''+L'')(Q+W)$.

Formula (28). $B=P(C''+L'')-(S+L'P)$.

From which it is evident that so long as the costs and losses of reduction exceed the costs and losses of sorting, the operation is advisable (neglecting outlay on sorting plant).

In Fig. 6 it is assumed that the areas A, B, C, D

* This assumes that reduction works run the same, sorting or not sorting, and that the value of the rejecta does not vary greatly. The right-hand number represents the costs and losses if there were no sorting.

represent the total ore sent to the surface from which $P(Q+W)$ is sorted out.

Evidently $P(Q+W) = P' W + P'' Q$, where P' is the percentage sorted out of the total *tonnage of waste*, and P'' the percentage of the total quartz sorted out.

Formula (29). $$P' = \frac{P(Q+W) - P'' Q}{W}.$$

Assuming, as in many cases, that the tonnage of quartz rejected is negligible,

Formula (30). $$P' = \frac{P(Q+W)}{W} = P\left(1 + \frac{Q}{W}\right),$$

hence the areas C and D have P', not P sorted out. The factor needed is the effective stoping thickness $(Q+W) - P' W$ at each section, if the really unpayable parts of the ground are to be determined.

For instance, if the flexibility of the stoping operations permit leaving the part shown between sections 205 and 215 (assuming of course a corresponding dip dimension), it would evidently be important to do this should they fall below the point set as profitable.

For illustration,

$$\text{Let } P = 20\ \% \text{ and } \left(\frac{Q}{W} + 1\right) = 2, \text{ then } P' = 40\ \%,$$

hence the thickness of waste will be reduced by this amount (*i.e.*, multiplied by ·60).

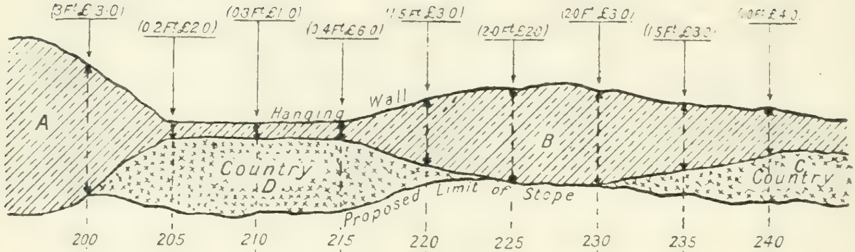
Formula (31). The stoping value S' will be

$$S' = \frac{v' t'}{t' + (1 - P')W},$$

where v' is the assay over t', and t' is the thickness sampled.

Table VI. sets out the calculations incidental to the continuous section system shown by Fig. 6, while Fig.

Fig. 6.


A
Hanging Wall
B
Country D
Country C
Proposed Limit of Stope
200
205
210
215
220
225
230
235
240

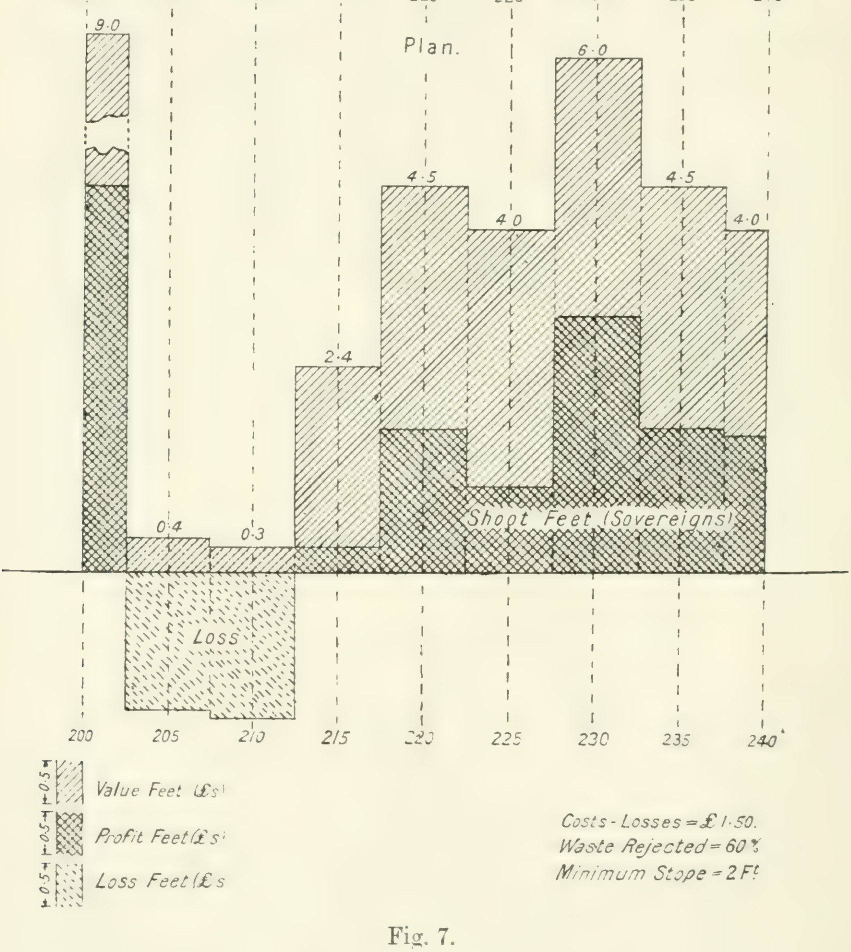

Plan.
9·0
0·4
0·3
2·4
4·5
4 0
6·0
4·5
4·0
Shoot Feet (Sovereigns)
Loss
200
205
210
215
225
230
235
240
Value Feet (£s)
Profit Feet (£s)
Loss Feet (£s)
Costs - Losses = £ 1·50.
Waste Rejected = 60%
Minimum Stope = 2 Ft

Fig. 7.

7 is the economic summary of the same whereby to finally determine the profit in this block, the periphery of which, so far as possible, should be sampled and reproduced in the above manner. It will be evident that all the data is given by the above figures whereby, not only to make such allowances as the mining conditions demand, but give the number both of the desired and the obtained sample sections wherewith to determine the risk-rate for the block.*

It will be seen that the shoot-feet calculation is but a weighting of the sample sections by the distance between them, and that the total profit in the block is :—

Formula (31*a*). $$P = F \frac{ld}{k(2l+2d)}$$

if the block is sampled on four sides, where F = number of sovereign shoot-feet, l = length of block, d = depth of block, and k = specific gravity constant.

* This assumes a knowledge of the "stoping factor" discussed under Premise V., and that all the drives and winzes are laid out like Figs. 6 and 7; should there be any breaks, these will increase the risk-rate for the block, as elsewhere dealt with.

TABLE VI.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
M Section.	Assay Value, v' in £.	Thickness of Sample in Feet $= t'$.	Value-feet, $v' t'$.	Thickness Waste Mined	Weighted Stoping Thickness, $t'(1-P')W$, Basis A.	Stoping Value, $\frac{v' t'}{t' + (1-P')W}$	* Stoping Profit per Ton.	* Profit-feet, Basis A.	* Profit-feet, Basis B.	Sovereign Shoot-feet, Basis A.	Sovereign Shoot-feet, Basis B
200. . .	3·00	3·0	9·0	0·0	3·0	3·0	+1·5	+4·5	4·5	+11·25	+11·25
205. . .	2·00	0·2	0·4	1·8	1·3	0·3	−1·2	−1·6	. .	−8·00	. .
210. . .	1·00	0·3	0·3	1·7	1·3	0·2	−1·3	−1·7	. .	−8·50	. .
215. . .	6·00	0·4	2·4	1·6	1·4	1·7	+0·2	+0·3	0·3	+1·50	+1·50
220. . .	3·00	1·5	4·5	0·5	1·9	2·4	+0·9	+1·7	1·7	+8·50	+8·50
225. . .	2·00	2·0	4·0	0·0	2·0	2·0	+0·5	+1·0	1·0	+5·00	+5·00
230. . .	3·00	2·0	6·0	0·0	2·0	3·0	+1·5	+3·0	3·0	+15·00	+15·00
235. . .	3·00	1·5	4·5	0·5	1·9	2·4	+0·9	+1·7	1·7	+8·50	+8·50
240. . .	4·00	1·0	4·0	1·0	1·6	2·5	+1·0	+1·6	1·6	+4·00	+4·00
Totals and averages,		. .	. .	. .	1·76	. .	+0·53	+0·93	1·8	+37·25	+53·75

* REMARKS.—Minimum stoping thickness, T = 2·0′, P at crusher = 20 per cent., P′ = 40 per cent.
Sorting percentages. Total costs and losses allowed = £1 10s. "Stoping factor" is taken at two sections.
To illustrate only, Basis B allows that the distance covered by Sections 205 and 210 will permit of leaving this ground unstoped.
Figure (6) allows 5 units of length between small Sections.

CHAPTER II.

BASES OF SOUND VALUATION.

When a report on a property is submitted, certain things are taken for granted, and while many of them have been discussed in several works, and in the proceedings of technical societies from time to time, they do not seem to have heretofore been set out *seriatim.*

The following is a list of the premises thought necessary to sound valuation, the first three having been discussed in the preface and introduction, while the remaining, bearing more especially upon sampling, follow in this chapter.

Premise I.—*A sense of economic proportion should be preserved even in examination work.**

Premise II.—*Modern reporting practice calls for a full presentation of data, calculations, and deductions therefrom.*

Premise III.—*Any investment implies the expectation of a return of the original capital,* 3 *per cent. annually to represent the rate received by investments conceived to involve a minimum risk, and a further rate of interest, commensurate with the risk and counting from the date when the investment was made.*

Corollary (*a*).—When dividends are delayed or suspended, those received must make good the loss due to such deferrence or suspension of interest.

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* This a more euphonious way of stating that common sense, on the ground as well as in choice of sampling and calculating systems, is a useful adjunct to accuracy.

Corollary (*b*).—As the interest during the deferred period may be regarded as a further investment of capital in the undertaking, interest on this at the risk-rate is to be expected; in other words, allowances for loss due to deferrence of dividends should regard the interest as compounding during the deferred period, thereafter reinvested at 3 per cent. annually.

Corollary (*c*).—As blocks of ore in a mine vary in tonnage and value-data (or number of sample of sections) available, as well as in time of exhaustion, estimates of their present value involve a consideration of "life," rate for risk, and deferrence for each.

Premise IV.—*Sound deductions as to value must be based upon mining operations as practised or practicable.*

Premise V.—*Sampling should conform to and clearly set out those variations in strike, thickness, colour, texture, parting, and pay, such as might be observed when stoping; as practised or practicable. These features to be shown, not only in cross-section, but in continuity of strike and dip.*

Premise VI. — *Cross-sectional measurement to be reduced to a line perpendicular to the dip of the vein.*

Premise VII.—*In each sample the pound-footage to be uniform.*

Premise VIII.—*Each unit of length in a sample to represent proportionate parts of the true vein thickness.*

Premise IX.—*The pound-footage to be determined by the distribution of the metal in the ore and by the breaking character of the surfaces sampled.*

Premise X.—*The spacing of sample sections should be governed by variations in sectional widths and values rather than by accessibility or convenience in calculation.*

Discussion of Premise IV.—*Sound deductions as to value to be based upon mining operations, as practised or practicable.*

As mining work is ordinarily divided into two classes, exploratory or development, and stoping or winning, so in sampling it is imperative to have these two operations in view. With reference to exploratory work; from its very nature we may deduce Premise V.; in fact, it is but a corollary of IV.

Discussion of Premise V.—*Sampling should conform to and clearly set out those variations in strike, thickness, colour, texture, parting, and pay, such as might be observed when stoping; as practised or practicable. These features to be shown, not only in cross-section, but in continuity of strike and dip.*

As mentioned above, Premise V. is really but a special form of Premise IV., the application of which calls for discussion.

Disappearance of Pay Streaks.—As the tendency of veins is to vary in strike, and when to this is added the sinuosities due to the vagaries of contractors on development work, a common result is that the "Pay" disappears behind either side of drives or raises; more particularly is this true of large deposits, especially where it is difficult to distinguish the ore from other vein-filling or wall rock.

True Block Values.—It frequently happens in the case of large veins, that there is both a foot and hanging pay streak, and when considerable lengths of, say, a drive expose only one or neither of these, the average value along that portion is evidently not a true index to that of the block above or below.

Variations in Stopes.—In practical stoping operations the constant effort is to widen or contract the stope in order to include available "pay" or to reject un-

payable parts and is one of the marks of sound underground practice; it must be taken as a premise, though not so stated.*

Stoping Practice the Guide to Sampling.—As in actual operations, the differences in texture, position of parting-slips and pay generally determine the lateral limits of the stope, likewise the same variations should govern the sampling across each vein section.

In practice, each mine has what may be called its "Stoping Factor" *—that is, the stopes can profitably be expanded or contracted in width and length within certain limits only, according to the ground, system of stoping, and methods of timbering and drilling. To illustrate; if "bunches" of unpayable ore with dimensions $10' \times 10' \times 1'$ were to exist on either wall, and machine drills were in use, it would probably be cheaper to stope than to leave this ore; if, on the other hand, these bodies were $40' \times 40' \times 3'$, and hand labour were employed, they would undoubtedly be left, especially if mining and treatment costs and losses were high.

Stoping Practice leads to "Continuous Section" Principles.—From the above it follows that, as in actual mining operations stopes are varied, having regard to

* This is referred to below as the *stoping* factor, the significance of which will be appreciated by those familiar with the wonderfully successful results obtained in Spanish countries by the ancient and much abused *coyote-ing* system. The more intelligent reader will see that Figs. 6 and 7, with Table VI., are nothing but the calculations incidental to counting the pros and cons of such a system. Many good mines have been needlessly closed through applying the wholesale idea to narrow veins; also, vast sums squandered in the purchase of rich but narrow veins only suited to the above system.

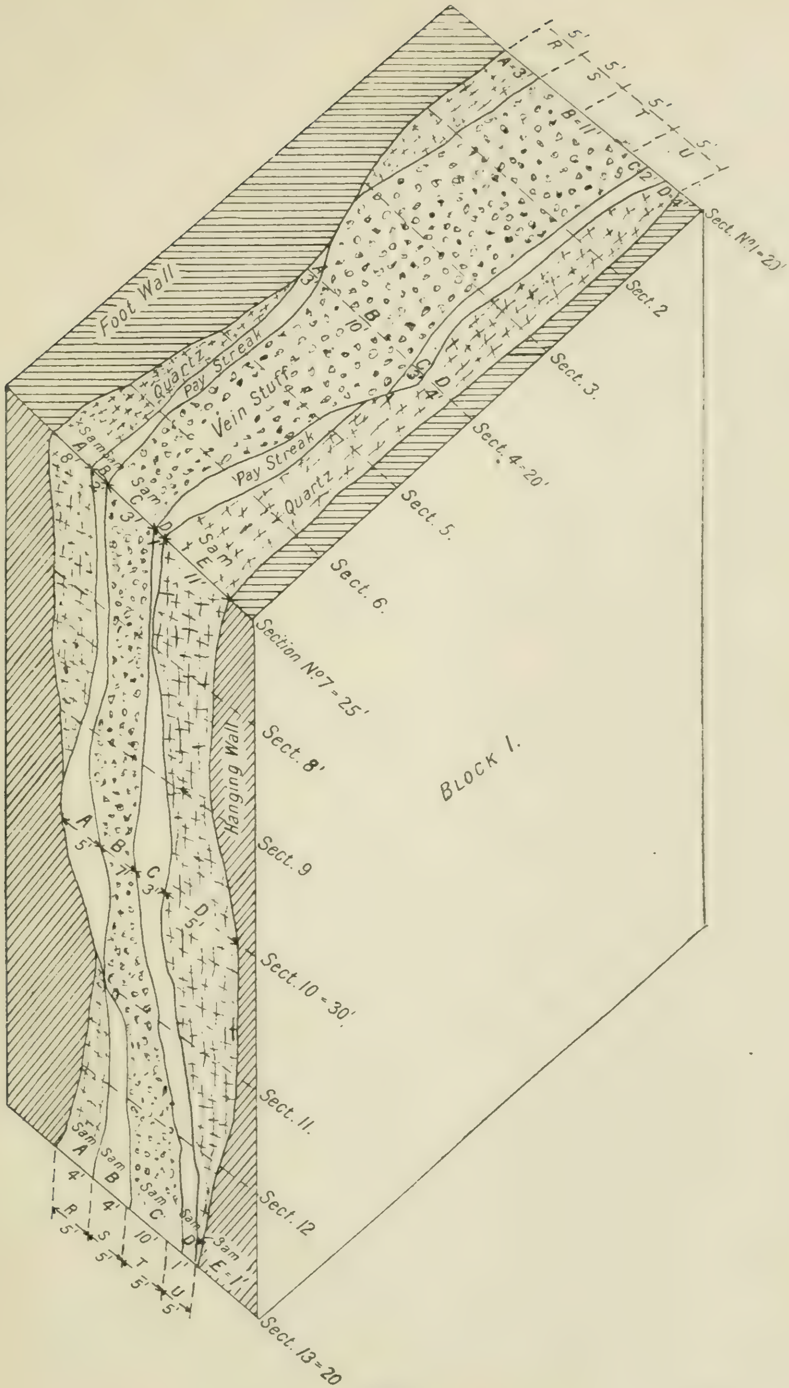

Foot Wall
Quartz
Pay Streak
Vein Stuff
Pay Streak
Quartz
Hanging Wall
Block I.
Sect. No. 1 = 20'
Sect. 2
Sect. 3.
Sect. 4 = 20'
Sect. 5.
Sect. 6.
Section No. 7 = 25'
Sect. 8'
Sect. 9
Sect. 10 = 30'
Sect. 11.
Sect. 12
Sect. 13 = 20

Fig. 8.

the dimensions of the payable of unpayable streaks, so, in order to intelligently calculate the value of a block of ground, sound sampling practice should allow for and record such variations in continuity or thickness as might be observed in stoping. We thus come to the continuous section system of sampling.

Fig. 8 will serve to illustrate, showing a block of ore such as occurs in the development of many veins.

Methods in Current Sampling Practice.—There may be said to be at least five methods in current sampling practice, each purporting to give the true sectional value at the points sampled. They are :—

(*a*) Cutting from wall to wall, thereby making one sample ;

(*b*) Dividing the section as projected on to the horizontal, into equal parts, making a sample of each ;

(*c*) Dividing the section haphazard, or as determined by convenience in sampling ;

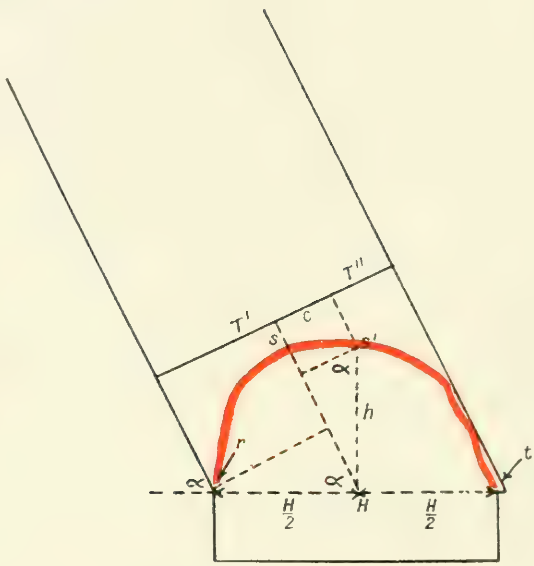
(*d*) Dividing the section according to the pay and parting streaks ;

(*e*) Dividing the section having regard to both stoping conditions and the angle of cut.

First Method (*a*) *of Cutting Samples.*—The practice of cutting from wall to wall in one sample (*a*) will not be dealt with, beyond stating that it would be permissible only if the vein were of uniform grade throughout the section, or the " back " presenting an unarched surface, conditions rarely met with.*

* The errors incurred by sampling in this manner will be seen by reading the discussion following.

To face p. 83.]



$$T' = \frac{H}{2} \sin \alpha + h \cos \alpha.$$

$$T'' = \frac{H}{2} \sin \alpha - h \cos \alpha.$$

$$\text{Error} = \pm h \cos \alpha \; [f \text{ (value } c)].$$

Fig. 9.

Second Method (b) of Cutting Samples.—With regard to (*b*)—that is, to dividing the projected section into equal parts, and making a sample of each—very serious errors occur, as dealt with under Premises V. and VI., but for the moment only those shown by a study of Fig. 9 will be touched on. This will serve to show the error incurred when an ordinary section is divided into two equal horizontal parts, separately assayed, and each multiplied by $\frac{H}{2}\sin\alpha$ to get the value feet. From this it will be seen that the assay of the arch *r*-*s*′ should be weighted by T′—*i.e.*, by $\frac{H}{2}\sin\alpha + h\cos\alpha$. Or, expressed differently, the part of the sample trench *s*-*s*′ will vitiate the true value of *r*-*s*′ accordingly as it be richer or poorer. In other words, if it also be expected to know in which part of the section the pay lies, this method of equal horizontal division will give rise to an error, which will be a function of the cosine of the dip of the vein (α); of the difference in height between the ends of the sample; and of the value of the ore between the samples. (See Fig. 9 and Table VII.)

TABLE VII.—Table of Errors through Horizontal Equi-Spacing of Samples in Sections.

Section No.	Cos (Dip 65).	Height above Horizontal (h).	h Cos 65°.	True Thickness.	Horizontal Width allowed by A.	Corrected by A, using Sin.	Error in Feet.	Error in per cent.
1,000	·4226	6·3	2·7	6·6	4·3	3·9	2·7	40
1,001	·4226	4·6	1·9	5·5	4·0	3·6	1·9	35
1,002	·4226	7·0	2·9	6·0	5·0	4·5	1·5	25
1,005	·4226	1·5	0·6	5·1	5·0	4·5	0·6	10
1,006	·4226	5·1	2·1	6·8	5·0	4·5	2·1	30

Table VII. shows the results of the study of five sections met with in recent practice, yet accepted as sound.

Turning to Fig. 8 and to Table VIII., the value of sections 1, 4, 7, 10, 13 is given as found by sampling according to pay and parting streaks, while Table IX. gives the results had from dividing the section into five-foot samples.

Assuming for the moment that the section permits of making the cut perpendicular to the dip of the vein, the sectional value-feet would be the same.* If, however, it be ceded that sampling is but a guide to probable profits, and that these are largely, if not principally, dependent upon the "pay" and "cost" features of the stoping operations, the errors of "Equal Division" should be evident. Not only this, but deductions from the data collected may not only be inadequate, but misleading, when applied to block valuations. For example, in Fig. 8 the quartz streak in the hanging sample D has, according to Tables VIII. and IX., a mean assay of $1.50, and is unpayable. By the equal division method the mean value of the hanging 5′ sample has, however, an assay value of $12.20. In other words, would pay to extract if a combination factor † of $9 were assigned in each case.

But the mean assay value of the whole block would show by this last method of sampling that it could be stoped only at a loss. A study of Fig. 8 and Table X.

* These calculations being for illustration purposes only, the effect of the corner and terminal samples are neglected.

† Costs plus losses.

TABLE VIII. (see Fig. 8).—BLOCK I.

Section.	Sample A.			Sample B.			Sample C.			Sample D.			Sample E.			Thick.	Total Value-feet.	Average Value.	Section Cost-loss-feet.	Section Profit-feet.
	Assay Value.	Thick.	Value-feet.	Assay Value.	Thick.	Value-feet.	Assay Value.	Thick.	Value feet.	Assay Value.	Thick.	Value-feet.	Assay Value.	Thick.	Value-feet.					
1, .	$11·0	3	33	2·0	11	22	$55	2	100	$1·5	4·0	6	..	..	..	20	171	8·5	180	— 9
4, .	20·0	3	60	1·5	10	15	100	3	300	2·5	4·0	10	..	..	..	20	384	19·2	180	+ 204
7, .	2·0	8	16	11·0	2	22	3	3	9	50·0	1·0	50	1	11	11	25	108	4·3	225	—117
10, .	10·0	5	50	2·0	7	14	15	3	45	1·0	15·0	15	..	..	..	30	123	4·1	270	—147
13, .	1·5	4	60	12·0	4	48	6	10	60	75·0	1·0	75	5	1	5	20	194	9·7	180	+ 14
Totals,	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	115 23	980	8·5	..	—55 *

* It should be clearly understood that as these sections are for illustration purposes only, the effects of the terminal samples and of Section 7 are neglected—*i.e.*, the shoot-feet basis is neglected, though shown in Fig. 7.

TABLE IX.—EQUAL SPACING OF SECTION INTO 5′ INTERVALS. (See Fig. 8.)

Section.	Sample R.			Sample S.			Sample T.			Sample U.			Sample V.			Sample W.			Total Value-feet.	Average Value.
	Assay Value.	Thick.	Value-feet.	Assay Value.	Thick.	Value-feet.	Assay Value.	Thick.	Value-feet.	Assay Value.	Thick.	Value-feet.	Assay Value.	Thick.	Value-feet.	Assay Value.	Thick.	Value-feet.		
1, .	7·4	5	37	2	5	10	12·6	5	63	12·2	5	61	..	..	..	..	..	..	171	
4, .	12·6	5	63	1·5	5	7·5	40·9	5	205·5	22	5	110	..	..	..	..	..	..	384	
7, .	2	5	10	5·6	5	28	12	5	60	1	5	5	1	5	5	..	..	..	108	
10, .	10	5	50	2	5	10	9·8	5	49	1	5	5	1	5	5	1	5	5	123	
13, .	3·6	5	18	9·6	5	48	6	5	30	19·6	5	98	..	..	..	..	..	..	194	
Total,	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	980	

clearly shows this to be another error for assuming that the stoping factor and physical conditions permitted, the quartz hanging streak would remain unstoped, thereby giving + 118 profit-feet, against − 55 by the equal division method.

Mistaken Block Calculations.—Summing up, one may say that sampling by (*b*), or the method of dividing the section into equal horizontal sample lengths, does not permit of block calculations based upon stoping conditions, and may lead to an error resulting in the rejection of a payable block.

Continuity of Section Imperative.—The assumption will be noticed in the above argument that we are in possession of the necessary data bearing upon continuity, though not necessarily indicated by Tables VIII., IX., and X. But are we ?

Referring to Fig. 8, it will be seen that the sections are taken at 10′ intervals ; also from Table VIII. that there are no adequate data whereby to recognise the same streak in different sections ; in other words, no record of continuity along the strike. But this very recognition is the *sine quâ non* of sound calculation, as based on effective stoping practice.

The tabular information being insufficient, this must be forthcoming from other data, to be had from a study of the faces themselves, and may be presented graphically in practically the form set out by Fig. 8, or may be indicated in the field books to be described later.

The essential point is to set out the continuity of each streak, and the writer emphasises his preference for the graphical presentation of factors governing stoping

TABLE X.—BLOCK I.—ANALYSIS.

Section.	Sample A.			Sample B.			Sample C.			Sample D.			Sample E.			Total Value-feet,	Total Profit-feet, Basis A.	Total Profit-feet, Basis B.	Stoping, Thickness Basis B.
	Value-feet.	Profit-feet, Basis A.	Profit-feet, Basis B.	Value-feet.	Profit-feet, Basis A.	Profit-feet, Basis B.	Value-feet.	Profit-feet, Basis A.	Profit-feet, Basis B.	Value-feet.	Profit-feet, Basis A.	Profit-feet, Basis B.	Value-feet.	Profit-feet, Basis A.	Profit-feet, Basis B.				
1, .	33	+ 6	+ 3	20	− 77	− 88	110	+ 92	+ 90	6	− 30	..	..	..	..	171	− 9	− 5	16
4, .	60	+33	+30	15	− 75	− 85	300	+270	+270	10	− 26	..	..	..	..	384	+204	+215	16
7, .	16	−56	−64	22	+ 4	+ 2	9	− 18	− 21	50	+ 41	+ 40	11·0	−88	..	108	−117	− 43	14
10, .	50	+ 5	− 0	14	− 49	− 56	45	+ 18	+ 15	15	−120	..	..	..	..	123	−147	− 41	15
13, .	6·0	−30	−36	48	+ 12	+ 8	60	− 30	− 40	75	+ 66	+ 65	5·0	− 4	−5	194	+ 14	− 8	19
Totals,	219	−42	−67	119	−186	−219	524	+334	−314	156	− 69	+105	16·0	−92	−5	980	− 55	+118 +1·5	16

Basis A assumes stoping from wall to wall, ... Combination factor of $9.0
Basis B assumes stoping of those samples marked *s*. . . . ,, ,, 10.0

practice as obtaining in each block, especially in examinations of importance, while perhaps favouring the book-entry method for stope face work in routine stope administration during operation.

Organisation to secure Administrative Efficiency.—As mentioned in the Preface, mine reporting organisation bearing upon administrative efficiency is wofully lacking, and this method of graphical record is found to be one of the strongest incentives to adequate underground study. Particularly in examinations, the tendency is to regard the work as of passing importance, wherein no check upon accuracy can be forthcoming, aside from resampling. But the Tiro himself realises that Nature gainsays a certain duplication of assay returns, even when resampling the same trench, with the result that the most slovenly work prevails and passes undetected; especially is this true if sample trenches be not carefully plotted and clearly shown with corresponding numbers on plan and in underground workings. Hence the graphical record is an invaluable adjunct to effective administrative control during examination work, for even those most indifferent to accuracy can scarce bring themselves to signing such a damning document as this presents.

Another equally important check upon administrative efficiency will be set out in the discussion of Premise VIII.

*Stoping Factor.**—Reverting to Fig. 8, it will be seen that were stoping conditions favourable, and with a stoping factor of, say, 20′, instead of the 30′ set out

* See also page 80.

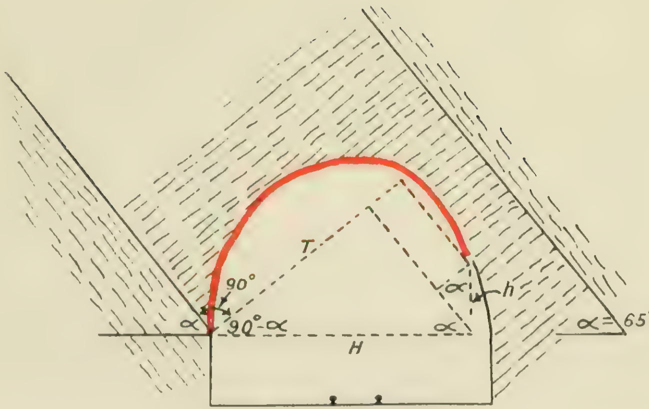
in the previous calculations, the body of ore shown on the foot-wall at sections 5, 6, 7, 8, and 9, as the A sample, could be left, to the further advantage of the profit-feet in the block. In fact, such a continuous section presents many features leading to intelligent study and sound conclusion, if the reporting engineer be really in touch with the technique of his subject. In practice, the records call for a delineation of the walls, parting-slips, and breaking character, such as govern underground operations ; details of this kind, however, will be supplied from the knowledge and ingenuity of the experienced, without which qualities the whole system would be unintelligible or unacceptable.

Third Method (*c*) *of Cutting Samples.*—With reference to (*c*), the method of dividing the section haphazard, or as found physically convenient, the same objections apply as in the case dealt with above.

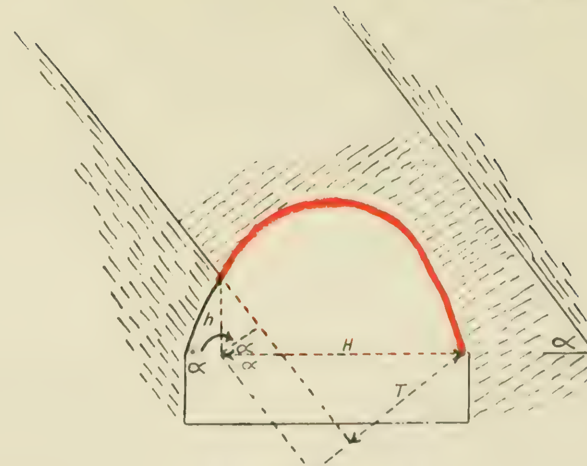
Fourth Method (*d*).—Concerning (*d*), the division of the section according to the pay-streaks and partings, but *without* the continuous section adjunct, it will appear upon a little study of Fig. 8 that the data are insufficient wherewith to certainly identify the same streak in the different sections. Yet upon this very feature hangs the sound valuation of the block as a whole, and upon these in turn must be based the appraisement of final value.

Fifth Method (*e*) *of Cutting Samples.*—With reference to (*e*), the method of sectional valuation based upon such variations of " pay " and physical features as might be observed in stoping practice, the principal points will have appeared from the foregoing discussion, while the angle of the cut will be dealt with under Premise VIII.

[*To face p.* 90.



$T = H \sin \alpha + h \cos \alpha.$



$T = H \sin \alpha - h \cos \alpha.$

$T = H \sin \alpha \pm h \cos \alpha.$

Figs. 10 and 11.

To amplify these points, however, that their application to and use in current practice may be quite clear, the following brief outline is offered :—*

Continuous Section System.—If one imagine a roll of tracing cloth fastened against and along the whole top of the drive, and to have traced upon it the veins, partings, and slip-faces, together with the sides of the drive ; if we now transfer this to the drawing table, and reduce it to a convenient scale, we have the principle of the continuous section system ; naturally in practice a scale sketch only is made, preferably on squared paper.

Of course, if feasible, measurements at right angles to the dip of the vein would be taken, a common practice being to measure out intervals of 10 feet, if the reefs be fairly uniform in thickness and value ; samples are then taken of each streak and separately assayed.

The widths of the streaks, partings, distances to sides of drives and slip-faces, to which the stope would naturally break, are taken at each of these points for plotting, washing down the faces or backs, if necessary, to be assured of the continuity.

Fig. 12 will perhaps serve to show this more clearly.

Let sub-figures A, B, C (Fig. 12) be cross-sections of the same drive at 10-foot distances apart.

Figures D, E, F will be the longitudinal sectional appearances of the same along the line A-B.

If, now, the intervening spaces in the drive between 1,500 feet, 1,510 feet, 1,520 feet be filled in as in Fig. G,

* See the "Continuous Section System of Mine Sampling" by the writer in the *Proceedings of the London Institute of Mining and Metallurgy* for December 19th, 1901.

we have a graphic office record of what reefs are followed, and where lost.

Fig. H shows the same sections as they appear in practice, where the outside * lines show the sides of the drive, and the short inner † lines the slip faces to which the rock would break in stoping.

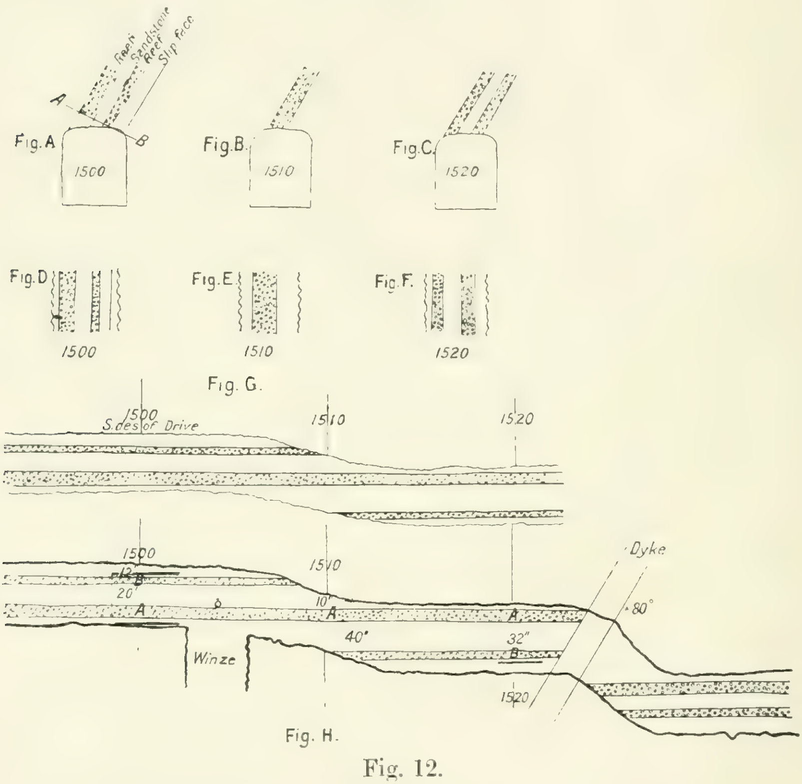


Fig. 12.

It is sometimes found simpler to put the dimensions of the partings on the sketch, and to designate the reefs by A, B, C, etc., with the particulars of those to one side.

* In practice these would be indicated by a blue pencilled line.

† This would be shown by a red line in the field book.

Advantages of Continuous-Section System.—One of the most powerful aids given by the continuous graphic system is the independence of mere value for the recognition and correlation of vein features.

As will be seen by these sketches, if the foot wall streak of Section 1,500 carry the pay, and it were lost before 1,510 were reached, the ordinary way of recording data would shed no light, and if another leader were picked up, the erroneous inference of a fault might easily be made, while were no "pay" met with thereafter, a valuable block might be condemned. Again, no calculations even approximating stoping conditions could be made without data similar to that shown, at least in cross-section.

Premise VI.— *Cross-sectional measurement to be reduced to a line perpendicular to the dip of the vein.*

Insomuch as block calculations entail estimates of cubic contents, this reduction is taken as imperative.

The Price of Carelessness.—The failure to adequately recognise the above premise led, as hereinafter mentioned, to errors which, if obtaining throughout the mine, would represent some £50,000 Sterling in an examination known to the writer (see Table XI.).

It would appear at first thought that so simple and so commonly accepted a Premise should need no enlarging upon, but, as will be shown, the haziest notions appear to obtain with regard to its practical application.

From Fig. 9 the true thickness of the vein $T' + T''$ is the horizontal distance (H) into sin α, where α is the dip of the reef.

So much will be granted by both old and new schools,

but in use, in systems aiming at the suitable application of theory, they appear to be at complete variance.

The figures numbered 14, 15, 16, 17, and 18 are reproductions to scale of sections met with in current practice, and are actual plotted results obtained by taking 1-foot off-sets from the horizontal to the sample trench, which is picked out in jagged red from *a* to *i*; these show the neglect of the above elementary premise, as well as of others.

TABLE XI.—TABLE OF ERRORS WHEN USING THE COSINE CORRECTION.

No. of Section.	*h* Difference in Height, Ends of Section.	*h* × Cos 65.	T True Thickness Vein.	Horizontal Meas.	Reported Thickness of Vein.	Error in Feet.	Error in Percentage.
1,000	3·0	1·3	13·1	13	11·8	1·3	− 10
1,001	2·2	0·9	8·2	8	7·3	0·9	− 10
1,002	1·1	0·5	9·5	10	9·1	0·4	− 4
1,005	1·0	0·4	14·0	15	13·6	0·4	− 3
1,006	3·2	1·3	14·9	15·0	13·6	1·3	− 9
250	5·4	2·3	14·0	13	11·8	2·2	− 15
151	2·7	−1·1	9·7	12	10·9 (?)	1·2	− 10

Premise VII.—*In each sample the pound-footage to be uniform.*

Uniformity of Cut.—By pound-footage is meant the number of pounds per linear foot of sample. In practice this is usually recognised by cutting a trench or groove of uniform depth and breadth; occasionally, also, where the arch may not be avoided, by broadening the cut at the centre to offset the tendency to greater depth at the ends of the sample. When a separate sample is made

To face p. 95.]

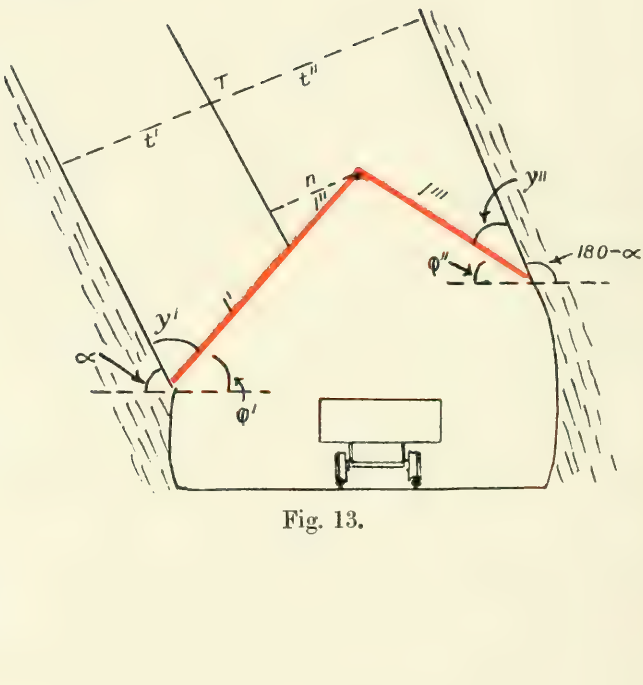


Fig. 13.

of each pay streak, care in this is evidently of less importance.

Premise VIII.—*Each unit of length in a sample should represent proportionate parts of the true vein thickness.*

To illustrate ; if the trench of a single sample measure 4 feet long, and the projection of the true vein thickness gives 2 feet, then each single foot of the sample must correspond to 6 inches in thickness. A phase of this is sometimes expressed as the " Terminal Sample Principle," indicating that the ends of the sample may not vary in slope from that of the centre.

This very manifest principle would appear to call for little discussion, and yet by Table XII. one sees that practice would have us believe that it may be neglected ; that errors of a thousand per cent. are of no practical import ; that systems correcting such mistakes may be relegated to visionaries.

Referring to Fig. 13, it is assumed that each streak (t' and t'') is uniform in value both across and for some distance along the dip, and that each contains different values.

Wanted V the average value across the true vein thickness T.

t' = true thickness of lower streak.

t'' = true thickness of upper streak.

l' = length of straight cut in lower streak.

l'' = length of other straight cut in upper streak.

l''' = length of other straight cut in same upper streak.

v' = true value, per ton, of t'.

v'' = true value, per ton, of t''.

A $=$ Assay value of whole sample cut, L.

L $= l' + l'' + l'''$.

$y'' = 180 - (\phi'' + 180 - \alpha)$

$= \alpha - \phi''$.

Referring to Fig. 14,

Formula (38). $\mathrm{V} = \dfrac{t'v' + t''v''}{\mathrm{T}}$.

Formula (39). $\mathrm{A} = \dfrac{l'v' + l''v'' + l'''v''}{\mathrm{L}}$.

E $=$ Error.

Formula (40). $\mathrm{E} = \dfrac{t'v' - t''v''}{\mathrm{T}} - \dfrac{l'v' + l''v'' + l'''v''}{\mathrm{L}}$.

If there be no error, $\mathrm{V} - \mathrm{A} = 0$ and $\mathrm{L}(t'v' + t''v'') - \mathrm{T}(l'v' + l''v'' + l'''v'') = 0$.

But from the figure, $\mathrm{T} = (l' \sin y') + (l'' \sin y') + (l''' \sin y'')$, and $l' = \dfrac{t'}{\sin y'}$; hence

Formula (41).

$$\mathrm{L}\,(t'v' + t''v'') - \mathrm{T}\left(\frac{t'v'}{\sin y'} + \frac{nv''}{\sin y'} + \frac{(t'' - n)v''}{\sin y'')}\right) = 0,$$

when $\sin y' = \sin y''$, $\mathrm{L} \sin y' = \mathrm{T}$; hence error $= 0$ only when $y' = y''$.

From the above it may be stated that a sample may not be carried around curved surfaces composed of streaks varying in value, without error, except when the cut makes equal angles with the parallel walls; the error would be a function of the slope of each component and the streak thickness, as well as of their values.

Table XII. will show the result of studying the six sections before referred to, and may be found interesting and perhaps instructive; at least with regard to what passes current as sound practice. The errors set out

were calculated on the assumption that if the sample l' (Fig. 13) were to be mixed with l' and l''', in order that there should be no error.

Formula (42). $$\frac{l'}{t'} = \frac{l''}{n} = \frac{l'''}{t'' - n}.$$

Formula (43). $$\text{Error} = \frac{l'}{t'} - \frac{l''}{t''}.$$

Formula (44). $$= \frac{l'}{t'} - \frac{l'''}{t'' - n}.$$

TABLE XII.—TABLE OF INTERNAL RATIO SAMPLE ERRORS.

1	2	3	4		
Section No.	Letters showing Portion of Sample.	Length of Portion of Sample. Feet.	Length Projection on to Line perpendicular to Dip.	Ratio of Columns 3 to 4.	Error in Percentage.
1,000	*g-h*	1·0	0·8	1·3*	260
1,000	*h-i*	1·4	0·3	4·7	
1,001	*a-b*	4·6	3·3	1·4	40
1,001	*b-c*	2·1	2·0	1·0*	
1,002	*a-a'*	2·6	0·2	13·0	1,200
1,002	*c-d*	2·8	2·7	1·0*	
1,005	*e-f*	1·4	1·3	1·0*	50
1,005	*f-g*	3·6	2·3	1·5	
1,006	*g-h*	3·6	1·8	2·0	100
1,006	*f-g*	1·7	1·6	1·0*	
1,007	*c-d*	2·3	1·8	1·3*	900
1,007	*d-e.*	1·3	0·1	13·0	

The deduction from the above analysis is that the

* These are taken as the *correct* ratio, and the error of *principle* as the ratio of the difference in ratios to the ratio marked with an asterisk. As seen by Formula (6), the error in *value* of the section is a function of the value and thickness of the pay streaks, as well as of the angles of cut.

sampling of curved surfaces should conform to the pay streaks—*i.e.*, each streak should be sampled separately, in order to arrive at the true sectional value. Practice, however, often shows a series of streaks, making up the arch, and of such a nature that stoping practice would compel their being brought down together.

According to Fig. 13, with the value of t' and t'' substituted, the value feet across such a section would be—
Formula (45). $TV = (l' \sin y'v') + (l'' \sin y'v'') + (l''' \sin y''v'')$ with as many members on the right as there are separate samples.

If the preceding reasoning be sound, then only by determining each factor of the above equation can the value feet of the section be arrived at. The length and assay values of each sample is obtained as in ordinary practice, but the measurement of y', y'' constitutes an innovation so far as the writer knows.

Automatic Elimination of several Errors.—Referring to Fig. 14, it will be clear that the inclination of l' from the horizontal can be taken by means of the spirit level or pendulum inclinometer, hence the value of ϕ' obtained; also from the same figure that $y' = [180 - (\alpha + \phi')]$.

Substituting this in (45), we have—
Formula (45*a*).

$$T\,V = l' \sin (\alpha + \phi')\, v' + l'' \sin (\alpha + \phi') v'' + l''' \sin (\alpha - \phi'')\, v'',$$

where α is the dip of the reef, and all the factors necessary to theoretically sound method are at hand.

In Table XIII. is shown the field-book expression of the above formula, the only additional work entailed, aside from sample calculation, being the measurement of the slope or dip of each sample, with a pendulum or

rule inclinometer,* an operation requiring but a few seconds of time, with the result that, not only is the method a precise one, but three important results are obtained ; these being—

Advantages of Closed-Traverse System of Sampling.—(*a*) *A very exact check on the measurements by reason of the "closed traverse" principle employed, hence an administrative feature of the first importance.*

(*b*) *The true thickness of the vein section being obtained, hence there is no possibility of the "cosine correction" errors of Tables* XI. *and* XII.

(*c*) *The true thickness of each streak being available, after the correlation of streaks is made, the data are in hand for the block calculations of the continuous section system.*

Tables XIV. and XV. show a tabulation of results intermediate between the field-book, Table XIII., and the Tables XVI., XVII., and XVIII. of the block calculations.

* The ordinary American carpenter's foot rule with spirit level and hinge joint graduated to degrees answers very well.

A table of sines, cosines, etc., will be found in the Appendix, for writing in the note-book should this be necessary.

TABLE XIII.—FIELD BOOK,* LEFT PAGE. SKETCH TO ACCOMPANY *en face* ON CO-ORDINATE PAPER.

No. of Section.	Letter of Sample.	l Length of Sample.	φ Dip of Sample from Horizontal.	Cos φ.	$l \cos \varphi$ Horizontal Thickness of Sample.	Horizontal Thickness of Section.		Error.	Dip of Vein from Horizontal.	Remarks.
						Calcul.	Taped.			
101	A,									Sample A corresponds with B of Sect. 100.
	B,									
	C,									
	D,									
	E,									
	F,									
	G,									
	H,									
	I,									
	J,									
	K,									
	L,									
	Total, .									

Mine, Level,

Sampler's signature, Date,

* NOTE.—American transit book usually used, they having not only tables of natural sines and cosines in back, but left page is in columns, while right is "squared" paper, and used for sketch of section.

1st Office Sheet.*

TABLE XIV.

No. of Section.	Letter of Sample.	l Length of Sample.	ϕ Dip of Sample from Horizontal.	y or $180 - (\alpha + \phi)$.	Sin y.	Actual Thickness Sample.	Actual Thickness Section.			Remarks.
	A,									
	B,									
	C,									
	D,									
	E,									
	F,									
	G,									
	H,									
	I,									
	J,									
	K,									
	L.									

Mine, Date Level,

Signature of sampler, Name of calculator,

* This sheet is kept by the head sampler, who checks each section taken by each man, and makes the continuous section sketch.

2ND OFFICE SHEET.*

TABLE XV.

No. of Section.	Letter of Sample.	True Thickness of Sample in Feet and Decimals.	Assay Number.	Assay Value of Sample.†	Value-Feet.	Profit per Ton.	P. Profit-Feet in Section. Basis A.	P. Profit-Feet in Section. Basis B.	Remarks.
	A,								
	B,								
	C,								
	D,								
	E,								
	F,								
	G,								
	H,								
	I,								
	J,								
	K,								
	L,								
	Totals,								

Combination Factors, Basis A =
,, ,, ,, B =

* This sheet is made up by the Engineer himself, having the continuous section before him, and should be signed.
† See signed Assay Sheet for values assigned each metal.

TABLE XVI.

	1	2	3	4	5	6	7
Block.	Area or Perimeter (to be stoped).	Height (H).	Length (L).	Average Thickness (T).	Minable Tons in Block.	Average Profit (P).	Profit in Block.

1. Comes from the continuous section plan. 2 and 3. Are from the Surveyor's signed plan.
6 and 7. Come from preceding tables which are plotted like Fig. 7, and are determined by planimeter or by calculation from tables. In the case of irregular spacing between the sample sections, these must, of course, be suitably weighted.

TABLE XVII.

1	2	3	4	5	6	7	8	9	10	11	11*a*	12	13	14	
Block.	Tons.	Life.	Deference.	Net Profit.	£ of Annual Profit.	No. of Sample Sections.	Sections Required.	Ratio 7 to 8.	r'''. Per cent.	Factor from Tables.	Factor from Tables.	Present Value. Basis A	Present Value. Basis B.	Present Value. Basis C.	Remarks.
A,	175,000	1	2	£182,000	£182,000	135	750	·18	25	..	..	..	..	..	..
B,	55,000	} 1 {	3	112,000	112,000	96	174	·52	10	..	..	..	..	..	..
C,	77,000		3	200,000	200,000	174	standard	..	5	..	..	..	..	..	..
D,	400,000	2	4	300,000	150,000	400	800	·50	10	..	..	..	..	..	..
E,	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Total,	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..

TABLE XVIII.

Block.	Tons.	Risk Rate Used.	On Plant. Basis I.						On Plant. Basis II.					
			Total Profit in Block.	Life of Block (n).	Number of Annual Pounds.	Delay in Mining (d).	Present Value, £1 per An.	Real Value, Block.	Total Profit in Block.	Life of Block (n).	Delay in Mining (d).	Present Value, £1 per An.	Number of Annual Pounds.	Real Value, Block.
I.,														
II.,														
III.,														

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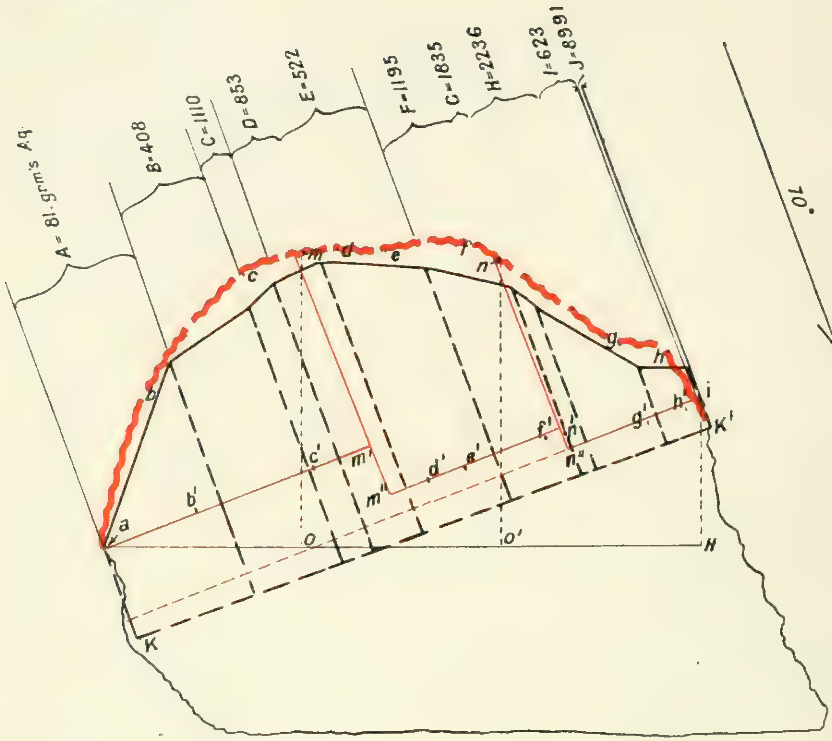


Sam.	Leng.	Proj.
A, .	4·2	2·6
B, .	2·0	2·0
C, .	·7	·7
D, .	1·3	1·0
E, .	2·3	2·0
F, .	1·7	1·6
G, .	·7	·3
H, .	2·5	1·6
I, .	1·0	1·0
J, .	·8	·1

Sam.	Leng.	Proj.
a-H, .	12·8	
K-K′,		12·9
a-*o*, .	42	
a-*m*′,		6·1
a-*o*′ .	4·2	
m″-*n*′,		3·9
O′-H,	4·2	
n″-*i*, .		2·9

Sam.	Leng.	Proj.
a-*b*-*m*, .	7·9	6·1
m-*n*, .	4·4	3·9
n-*i*, .	5·6	2·9
c-*d*, .	2·1	2·1
f-*g*, .	3·7	2·1
h-*i*, .	1·4	0·4

Fig. 14.

CHAPTER III.

SAMPLING.

Sampling Lodes.—If Figs. 14, 15, 16, 17, and 18 be turned to, sections taken from the examination beforementioned will be seen reproduced to scale, but before beginning the discussion of these reproductions from practice, it may be worth while to emphasise the difference between the sampling of small and large veins; with the former the dip is usually well before one, while the latter often implies working in difficult places and over irregular contours, with the result that a sample may easily run along a surface with nearly the same inclination as the vein without the fact being noticed. This is one reason that the closed traverse system of measurements is of so much value when dealing with large bodies with banded pay, the errors being corrected automatically.

Sampling Narrow Veins.*—On the other hand, the sampling of veins smaller than the stoping width present their own peculiar sources of error because of the inevitable admixture of waste in stoping, a portion of which only can be sorted out, whether in the mine or on the surface. As will be seen here also, the continuous graphic system of recording field data would seem to be the only one by means of which a consulting engineer can

* See also page 70.

feel sure that sound principles have governed the field work.

Figs. 6 and 7, with Table VI., illustrate such a continuous section and attendant calculations, as applied to small veins.

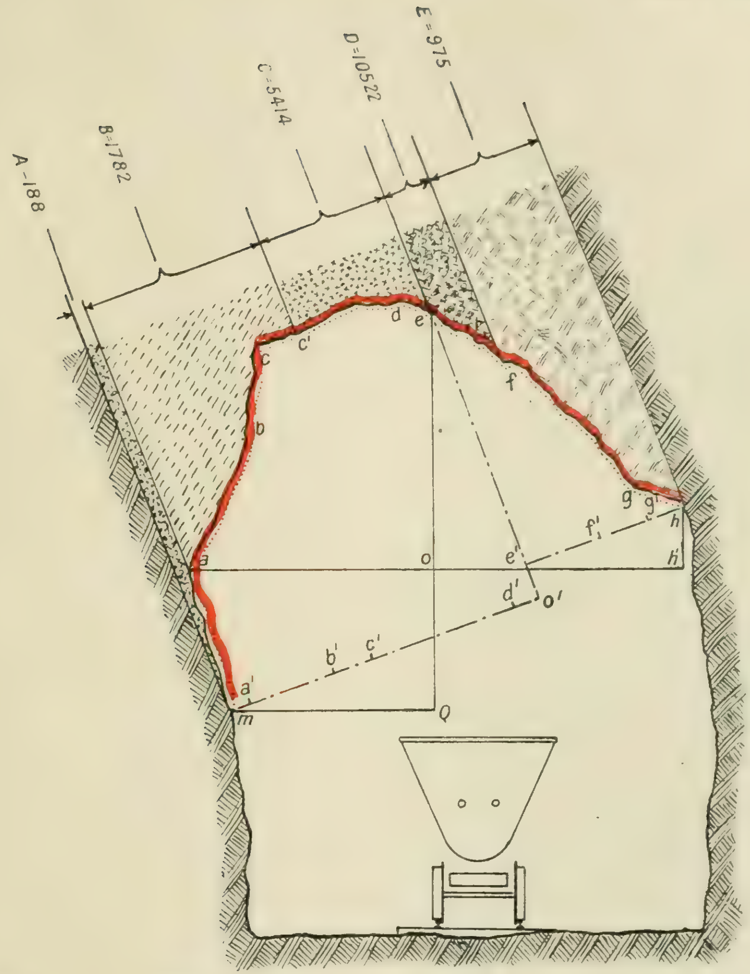
Discussion of Fig. 15.—Referring to Fig. 15, the broken black lines show the division of the section into two equal parts, the sampled surface being in red, *a′-e* making one sample, and *e-h* the other.

It will be seen that *a′-a*, running with the dip of the vein, was mixed with *a-b*, *b-c*, and *c-c′*, *c′-d*, and *d-e*, with the result that, according to the analysis of the figure, we have a theoretical error of 1,200 per cent. when comparing the projections of *a′-a* and *c-e*. In the succeeding examination this sample trench was re-sampled according to the stoping practice system, with results as shown in the figure, the silver contents being expressed in grammes.

As seen by Fig. 15, the true thickness of the foot-wall sample is *m-o′*, or 5·8, while that reported was *a-o* into the cos of 65°, or 4 feet, an error of 35 per cent. Here again the horizontal width was corrected by cosin *reduction*, instead of being increased as demanded by the section contour shown.

To automatically annul errors of both types, the "angle of cut" method, or closed-traverse system, was devised, and by it such a sample as *m-a* can be taken with perfect propriety, and yet be correctly weighted in the calculation.

It will be seen that sample B in the contour *a-b-c-c′* (Fig. 15) was taken in the second examination in one cut,

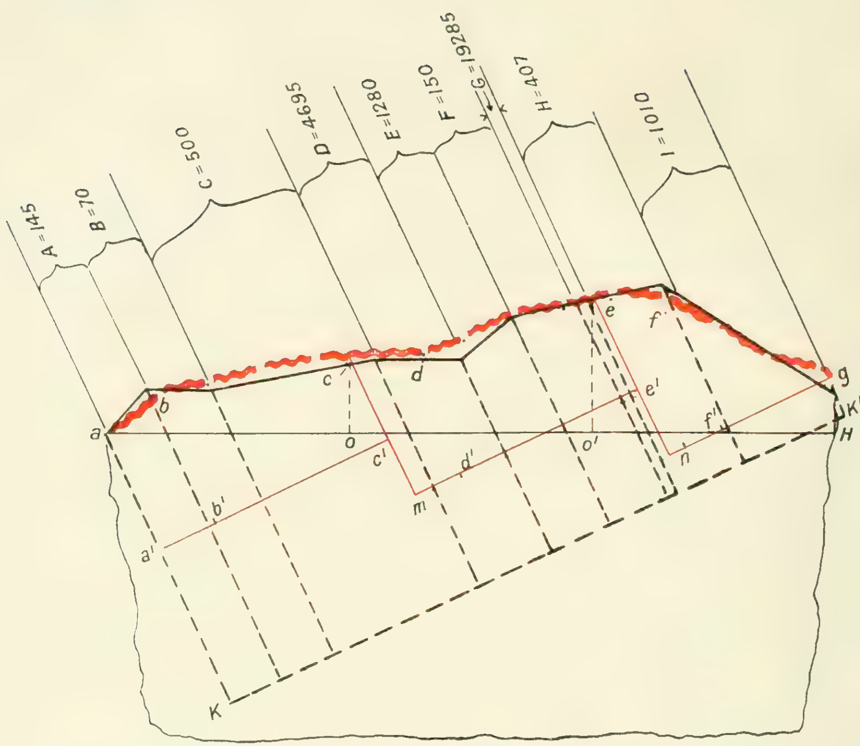


No. Sam.	Leng.	Sam. Proj.	No. Sam.	Leng.	Sam. Proj.	No. Sam.	Leng.	Sam. Proj.
A,	2·66	0·20	*a'-o*,	2·6	0·20	*g-h*,	0·8	0·7
B,	4·50	3·25	*a-b*,	2·5	1·7	*a'-o'*,	5·80	
C,	2·33	2·20	*b-c*,	1·8	0·9	*a-h'*,	8·80	
D,	1·20	1·00	*c-d*,	2·7	2·7	*m-o'* + *e'-h*,		8·85
E,	4·50	2·20	*d-f*,	2·4	1·5	*m-o'*	6·0	
a c-e,	10·0	5·85	*f-g*,	2·9	1·15	*m-Q*,	3·6	
e-f-h,	5·7	3·0						

Heavy red lines = sample trench or cut, divided into two samples, equal *on the horizontal.*

Fig. 15.

To face p. 107.]



No. Sam.	Leng.	Sam. Proj.	No. Sam.	Leng.	Sam. Proj.	No. Sam.	Leng.	Sam. Proj.
A,	1·2	1·1	*a*-H,	14·4		*a'*-*b*-*c*,	5·1	5·0
B,	1·3	1·2	K K'		13·4	*c*-*d*-*e*,	4·9	4·8
C,	3·3	3·2	*a*-*o*,	4·8		*e*-*f*-*g*,	5·0	3·6
D,	1·7	1·6	*a*-*c'*,		5·0	*d*-*e*,	3·8	
E,	1·3	1·3	*o*-*o'*,	4·8		*d'*-*e'*,		3·7
F,	1·3	1·3	*m*-*e'*,		4·8	*f*-*g*,	3·6	
G,	0·2	0·2	*o'*-H,	4·8		*f'*-*g*,		2·3
H,	1·5	1 5	*n*-*g*,		3·6			
I,	4·0	2·2						

Heavy red lines = sample trench or cut, divided into three samples, equal *on the horizontal.*

Fig. 16.

which could be justified only by the supposition that the ore along this part of the section was of even tenour.* The fact that it was uniform from a mechanical point of view would not warrant such practice unless sure of the uniformity in grade from *a* to *c′*. Where a streak is of considerable thickness, as, say, 3 feet or more, it would seem safer at the outset to sample having regard to the contour. For instance, in the case of Sample B, Fig. 15, it were better to make a sample each of *a-b*, *b-c*, *e-c′* until fairly certain of the characteristics indicating an even grade, when such a sampling as B might be thoroughly sound practice, but only in that event.

Fig. 16, the reproduction of sample trench 1,005 shows the small error of 3 per cent. in the estimate of the vein thickness, the height L being small.

On the other hand, the surface *a-b-c* taken in one sample was given only *a-o* sin 65, when entitled to *a′-c′* or *a-o* (sin 65°) + (*o-c* cos 65°), an error of over 10 per cent. Again, in sampling, *e-f-g*, *e-f* was mixed with *f-g*, which, according to the principles set out in the analysis, gives an error of 50 per cent.

The value of the streaks marked A, B, and C should be carefully noticed ; for taking the combination factor at $10.8, and the silver at $2 per 100 grammes, streaks A, B, C, running $2.90, $1.40, and $10, would evidently not pay to extract, and their aggregate thickness being 5·5 feet, they could probably be left, thus saving the costs of stoping and treating, while bringing up the grade of D, E, F, G, H, I correspondingly. Before this could be definitely calculated, however, the continuity

* See discussion of Fig. 14.

into the adjoining sections should be assured, the whole being preferably plotted in continuous section. If, then, the stoping factor were taken at, say, 20 feet vertically and longitudinally, also at, say, 1 foot latterally, and the continuity of the streaks established over three sections (20 feet), the corrections due to these features should enter into the block calculations.

The following tabular analyses would show that by sampling according to stoping practice, the value of the section could be made some 6 per cent. higher.

TABLE XIX.—Applying to Fig. 16.

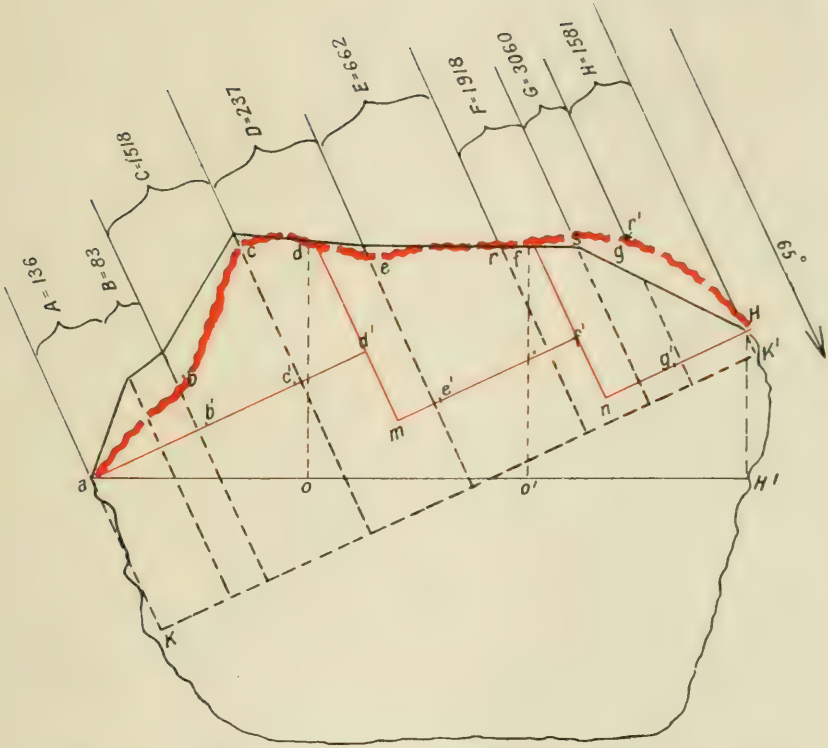
Sample.	Thickness.	Assay Value.	Value-feet.	Profit-feet, Basis A.	Profit-feet, Basis B.
A, . .	1·1	2·9	3·2	8·6	. .
B, . .	1·2	1·4	1·7	11·3	. .
C, . .	3·2	10·0	32·0	2·6	. .
D, . .	1·6	93·9	1·2	126·1	126·1
E, . .	1·3	25·6	33·3	19·3	19·3
F, . .	1·3	3·0	3·9	10·1	10·1
G, . .	0·2	385·7	77·1	74·9	74·9
H, . .	1·5	1·1	201·5	188·3	185·3
I, . .	2·2	20·2	44·4	20·7	20·7
Totals, .	13·6	39·7	537·7	390	416·3

With both Bases A and B, Combination Factor = 10·8.

Discussion of Fig. 17.—Turning to Fig. 17, Section No. 1,006, we find a sectional error of 9 per cent., while the foot-wall sample *a-b-c-d* was allowed *a-o* sin 65 for the thickness, instead of *a-d′*, or (*a-o* sin 65) + (*o-d* cos 65) —that is, an error of over 30 per cent.

Looking more closely into this sample, it will be seen that the richer streaks *b-c*, or C figure, are 40 per cent. out of proportion, as compared with *c-d*.

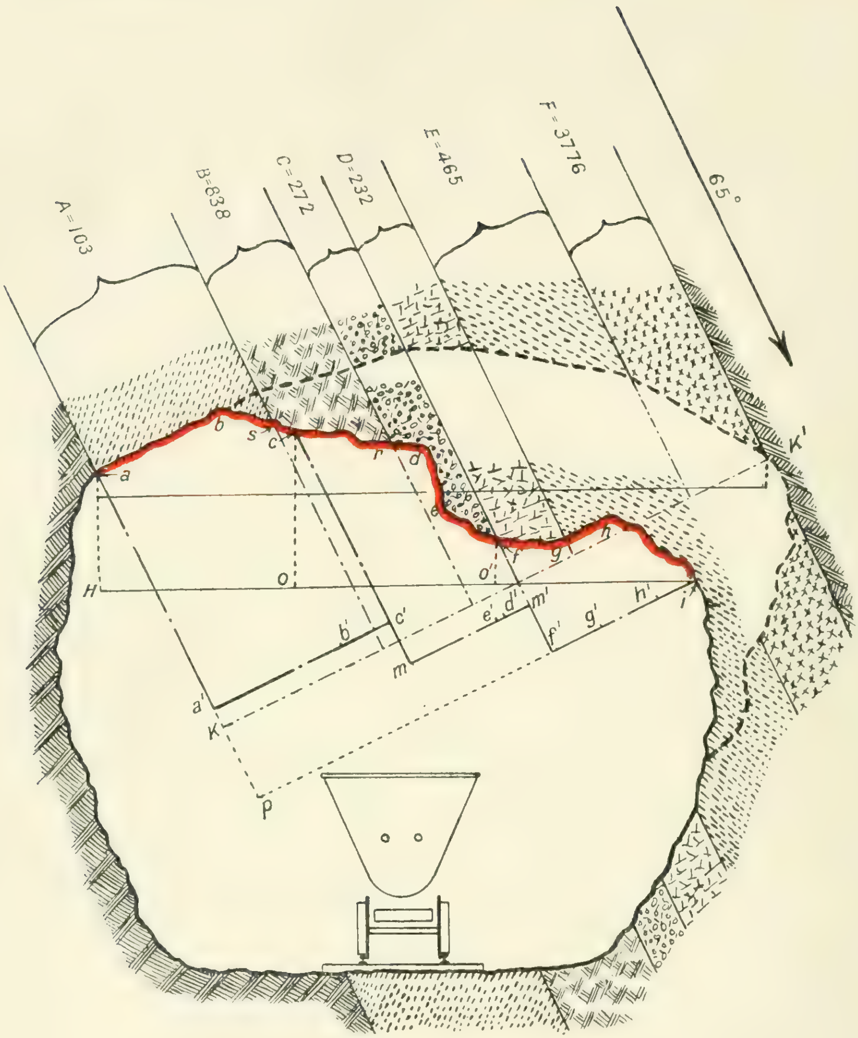
[*To face p.* 108.



No. Sam.	Leng.	Sam. Proj.	No. Sam.	Leng.	Sam. Proj.	No. Sam.	Leng.	Sam. Proj.
A,	2·3	1·6	*a-b*,	3·0	2·7	*a-d′*,		6·3
B,	0·9	0·9	*b-c*,	3·0	2·1	*o-o′*,	4·63	
C,	3·0	2·4	*c-d*,	1·6	1·5	*m-f″*,		4·3
D,	2·7	2·4	*d-e*,	1·4	1·1	*o′*-H,	4·63	
E,	2·9	2·7	*e-f*,	3·3	3·2	*n-h*,		3·4
F,	1·7	1·5	*f-g*,	1·7	1·6	*l-r*,	2·8	2·7
G,	1·6	1·0	*g-h*,	3·6	1·8	*r-f*,	0·6	0·6
H,	2·2	1·5	*o*-H,	14·0		*f-s*,	0·7	0·8
a-b-c-d,	7·6	6·3	K-K′,		14·0	*s-g″*,	1·1	1·0
d-e-f,	4·7	4·3	*a-o*,	4·63		*g″-h*,	3·2	1·5
f-g-h,	5·3	3·4						

Heavy red lines = sample trench or cut, divided into three samples.

Fig. 17.



No. Sam.	Leng	Sam. Proj.	No. Sam.	Leng.	Sam. Proj.		
A,	3·1	3·1	*a-b*,	2·5	2·5	H-*o* = *o-o'* = *o-i* = 3·6	
B,	1·8	1·7	*b-c*,	1·3	1·0	H-K'	12·1
C,	1·0	1·0	*c-d*,	2·3	1·8	H-*i*,	10·8
D,	1·2	1·1	*d-e*,	1·3	0·1	*p-i*,	8·7
E,	2·85	2·3	*e-f*,	1·0	0·5	H-K',	10·9
F,	2·85	1·7	*f-g*,	1·1	1·0	*c-r* = 1·8 (1·3)	
a-b-c,	3·8	3·5	*g-h*,	1·1	1·1	*c-f* = 9·5 (2·3)	
c-f,	4·6	2·3	*h-i*,	1·8	0·8	*s-c* = 0·4 (0·4)	
f-i,	4·0	2·9					

Heavy red lines = sample trench or cut, divided into three samples.

Fig. 18.

Sample *f-g-h* will also show that, aside from the irrational dimensional weighting of *g-h* in the mixing, the very rich sample F, G, H is weighted as O′H sin 65 when entitled to only *n-h*, or an error of 23 per cent.

By tabular analysis we would find as follows :—

TABLE XVI.—Referring to Fig. 17.

Sample.	Thickness.	Assay Value.	Value-feet.	Profit-feet, Basis A.	Profit-feet, Basis B.
A, . .	1·6	2·7	4·3	13·0	..
B, . .	·9	1·6	1·4	8·3	..
C, . .	2·4	30·4	73·0	47·1	47·1
D, . .	2·4	4·7	11·3	14·6	14·6
E, . .	2·7	13·2	35·6	6·4	6·4
F, . .	1·5	38·4	57·6	41·4	41·4
G, . .	1·0	61·2	61·2	50·4	50·4
H, . .	1·5	31·6	47·4	31·2	31·2
Totals, .	14·0	20·8	291·8	139·8	161·1

This table, being for illustration only, the Combination Factor used for Bases A and B is the same—$10·80.

Discussion of Fig. 18.—Turning to Fig. 18 (No. 1,007), one meets the same error in estimating the true vein thickness, only it is plus, corresponding to that shown in Fig. 11, being also $-h \cos \alpha$, and amounting to 11 per cent.

Even a casual inspection of the sample trench *c-r-d-e-f* will show the great error due to mixing *c-r*, representing 1·8 feet of sample (and 1·3 of vein) with *r-d-e-f*, representing 2·8 feet of sample, and only 0·9 foot of vein.

Again, the richer part of the section is given a dimensional weight of *O-O′*, while entitled to only *m-m′*—*i.e.*, an error of over 40 per cent.

After the sample trench *a-b-d-h-i'* was measured, stoping above the level was begun, and, when resampled, had the outline shown by the green lines, exposing the hanging-wall pay streak.

By the ordinary system of sampling, the true value of the whole section would not be even approximated, the effect being to reduce the real value in proportion as the section shown in red was lower than the mean value of the block. By the continuous section system, however, the passing of the hanging-wall streak into the side of the level at other points would be noted and plotted, while in the block calculations a value would probably be given it, as indicated by those of the nearer samples in the same streak.

This loss of pay is a common occurrence in all exploration work on big veins, and is unavoidable, insomuch as development drives are seldom as wide as the vein. The ordinary practice is, of course, to put in cross-cuts from time to time, something the reporting engineer should insist on having done whenever in serious doubt. This is often resented by the management, but if the owners be made to realise that it is altogether in favour of themselves much can be effected. By the graphical continuous system the consulting engineer has the data whereupon to telegraph, if necessary, for further work to be done at any point pending another examination.

It may be contended that an assay plan giving values only would serve the same purpose. If, however, the streak B in Fig. 18 ran sufficiently high to bring up the stoping value of the section to pay, it would not be suspected that the hanging streak had been left. Again,

even if the lower values gave cause to suspect some such occurrence, it could not be definitely known in the office whether the lost pay streak were in the foot or the hanging.

This section (Fig. 18) also indicates that the foot-wall streak could be very profitably left behind, providing, of course, that the adjoining section were to show similar occurrences, and the stoping factor permitted.

Premise IX.—*The pound-footage to be determined by the distribution of the metal in the ore, and by the breaking character of the surfaces sampled.*

Cross-Sectional Area of Cut.—This means that, if the breaking character be equal, one can with equal probability of error take samples of smaller pound-footage when sampling, say, iron ores, than when sampling gold quartz.

It also means, other things being equal, that one can more safely take long trenches of small cross-sectional area when sampling, say, soft sandstone reefs, than when working with "blocky ground."

It should be evident that, as the distribution of values in gold and silver ores are more irregular than most others, the weight per foot of sample trench should be greater—*i.e.*, is a function of the value of the metallic content.

In general, the writer stipulates for not less than 20 lbs. weight per running foot when sampling gold ores, and more if the ground be "blocky."

Premise X.—*Spacing or sample sections should be governed by variations in sectional widths and value, rather than by convenience in calculation.*

In other words, were we sampling a "patchy" vein of gold quartz with marked variations in thickness, these features should determine the distance between the sections, and not the desire to avoid arithmetical calculations.

Fig. 19 will serve to illustrate a not uncommon section, particularly in veins which conform to the bedding of argilacious rocks, though, of course, met with in other types of deposits.

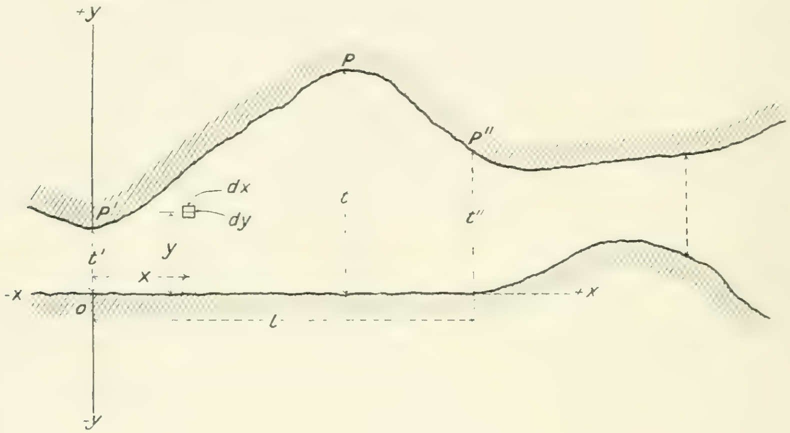


Fig. 19.

Discussion of Fig. 19.—Two sources of error arise from a regular spacing of sample sections, especially when they fall in a manner illustrated by the above figure. The two errors refer—one to the calculation of the area of the horizontal reef section, and the other to the estimates of the metal in the reef; the plus error of the area between two sample sections may compensate for the minus error in area between two others, as may the errors in value; or, again, one set of errors decrease the other, but this should not be counted upon.

In practice, where there is marked variations in thickness. the sounder procedure is to space the samples with reference to such changes, especially where preliminary assays indicate great variation in value, afterwards taking intermediate sample sections.

Referring to Fig. 19, the magnitude of the errors may be arrived at as follows :—

Let v = The value of the metal if the small area dy dx were sampled.

V = The average value over the area o-P′-P-P″-x'-o.

y = The vertical height of the sample area above o, which for convenience is taken as being a horizontal line.

x = The horizontal distance of the small area from the y axis.

t' = The thickness of the vein at o.

t'' = The thickness of the vein at x'.

v' = The assay value found by sampling over o-P′.

v'' = The assay value found by sampling over x'-p''.

a = Constant.

k = Constant.

By the ordinary calculus—

Formula (46).
$$V = \frac{\int_0^l \int_0^{f(x)} v\,dx\,dy}{\int_0^l \int_0^{f(x)} dx\,dy}.$$

The ordinary assumption that the mean of the values found at t' and t'' is the average value between them,

implies that the value follows the straight line law, or

$$v = a + k\,x = v' + \frac{v'' - v'}{l}\,x.$$

Likewise, the average thickness is usually taken at $\frac{t' + t''}{2}$, and the area at $l\left(\frac{t' + t''}{2}\right)$, whereas, as will be seen from the figure, the true area is

$$\int_0^l \int_0^{f(x)} dx\,dy = \int_0^l fx\,dx.$$

From the figure alone it will be clear that, in order to substitute a straight line law for the complex curve of the walls, the distance l must be such that the portion of the wall from P′-P and P-P″ will be fairly straight, in order to secure even an approximation.

With the cuts taken at P′ and P″, and calculated in the ordinary way, all the area P′-P-P″-P′ would be missing from the denominator of Formula (46), and V be correspondingly too high, assuming, of course, that $\int_0^l \int_0^{f(x)} v dx\,dx$ were given its true value between o-P-P′-P″-x'-o.

While samples may be spaced with reference to the contour of the walls, thereby securing an approximation to a straight line, sample cuts may not be arranged so as to escape the danger that the value in a cut taken, say, at P shall prove higher or lower than that found either at P′ or P″.

On this account we must say that, failing other data, the law of probability would have us postulate that the value will vary as—

Formula (47). $$v = a + kx = v' + \frac{v'' - v'}{l}\,x = \frac{(v'' - v')x + v'l}{l},$$

also

Formula (48). $$y = c + mx = t' + \frac{t'' - t'}{l}\,x = \frac{(t'' - t')\,x + t'l}{l}.$$

Substituting these in Formula (46)—

Formula (49). $$V = \frac{\int_0^l \frac{(t'' - t')\,x + l\,t'}{l}\,dx\,\frac{(v'' - v')\,x + l\,v'}{l}}{\int_0^l \frac{(t'' - t')\,x + l\,t'}{l}\,dx}$$

Formula (50). $$= \frac{l}{6}\,\frac{(2\,v''t'' + v''t' + v't'' + 2\,v't')}{l\left(\frac{t'' + t'}{2}\right)}.*$$

By the ordinary calculations—

Formula (51). $$V = \frac{t'v' + t''v''}{t' + t''},$$

hence the difference between Formula (50) and Formula (51) will be the ordinary error, E.

Formula (52). $$E = \frac{t''\,(v' - v'') + t'\,(v'' - v')}{3\,(t' + t''},$$

and this will equal *o* only when

$$t'' - t' = o, \text{ that is, when } t'' = t',$$

or when $$v'' - v' = o, \text{ that is, when } v'' = v'.$$

In other words, the ordinary calculations of value introduce an error whenever the vein varies both in thickness and metal contents at the two points sampled.

Whether the cost and delay incidental to the calculations of Formula (50) may be justified must be referred to our friend economic proportion, of Premise I., and Formula (52) will perhaps be referred to her.

* An excellent presentation of this same principle using the graphics of three dimensions will be found in an article by Mr. Crosley in the *Transactions of the I. M. M.* of some years ago.

CHAPTER IV.

EXPLANATION OF THE SINKING-FUND TABLE (XXII.).

THE ordinary tables are expressed in terms of unity—that is, the decimal parts of one which are to be set aside each year, in order to redeem unit capital (C) at the expiration of a given term (n). As the whole effort of the book is to identify the engineer with economics and finance, the table has been expressed in terms of percentages, the ordinary enquiry being, How many per cent. must be put aside each year to redeem an original outlay of one pound (or dollar)? Evidently this is merely a matter of pointing off, and the entries in this table will be found to correspond with those of other tables if they be pointed off two to the left.

In accordance with the general principle of defining risks, and the belief that in general those securities accepted as standard, bearing 3 per cent. per annum, and selling at par, a sinking fund compounding annually at 3 per cent. is the only one dealt with.

The table is also of use in solving those formulas which involve the expression r'' or $\frac{r'}{R - 1}$, the table being $\frac{\cdot 03}{(1 - \cdot 03) - 1}$.

Example.—Shares in a mine have been purchased at 30s., which yield 30 per cent. per annum, the life of the mine being taken at 12 years. What must be the

amount of yearly interest left after setting aside wherewith to redeem the capital? The shares yield 30 per cent. on one pound, or $\frac{30}{1\cdot5}$ on 30s.—*i.e.*, 20 per cent. on the investment. Looking at the left-hand column marked "Life in Years (n)," against 12 years will be found 7·05 per cent., which, taken from 20 per cent.,

TABLE XXII. — SINKING FUND (r''), OR MORE SENSIBLE SHARE-GAMBLER'S TABLE. $r'' = \frac{r'}{R^n - 1}$. (See also Table XXIII., Col. 6.)

Life in Years (n).	r'' or Rate Investment must yield in order to redeem Unit Capital in n years.		
	Redemption at 3 per cent.	Redemption at 4 per cent.	Redemption at 5 per cent.
1,	100·0000	100·0000	100·0000
2,	49·2611	49·0196	48·7805
3,	32·3530	32·0349	31·7209
4,	23·9027	23·5490	23·2012
5,	18·8355	18·4627	18·0975
6,	15·4598	15·0762	14·7017
7,	13·0506	12·6610	12·2820
8,	11·2456	10·8528	10·4722
9,	9·8434	9·4493	9·0690
10,	8·7231	8·3291	7·9505
11,	7·8077	7·4149	7·0389
12,	7·0462	6·6552	6·2825
13,	6·4030	6·0144	5·6456
14,	5·8526	5·4669	5·1024
15,	5·3767	4·9941	4·6342
16,	4·9611	4·5820	4·2270
17,	4·5953	4·2199	3·8699
18,	4·2709	3·8993	3·5546
19,	3·9814	3·6139	3·2745
20,	3·7216	3·3582	3·0243
25,	2·7428	2·4012	2·0952
30,	2·1019	1·7830	1·5051
35,	1·6539	1·3577	1·1072
40,	1·3262	1·0523	0·8278
45,	1·0785	0·8262	0·6262
50,	0·8866	0·6550	0·4777

For basis of table and formula, see pp. 24, 25.

leaves, say, 12·9 per cent. as the real return on the investment.

Basis of Table XXIV.*—This will be found set out in Formulas (10), (11), (12), and (13) of the Introduction.

Whenever the purchase price and other outlay on a mine exceeds the present value of the reserves, the difference is the price paid for the "possibilities" of a mine. As the life of many mines is either not published, or may not be even approximated, it is of great importance to know how many years' life this excess payment represents, for in far the majority of cases it is inadvisable to postulate a life very much greater than that of the reserves.

Evidently when this excess payment represents a certain life, the yearly tonnage mined, with the economic dimensions of the shoot, will give the depth which this payment really means ; this, of course, allows no deferrence. For this last see Tables XXVII. to XLII., and Example, which illustrate their use.

Example.—The dividends from a mine being, and likely to remain at, 30 per cent. per annum with the shares at £1·5, and the life of the reserves 2 years, how many years' life below must be assumed if one allow a uniform risk-rate of 10 per cent. ?

The yield on the capital invested will be $\frac{30}{1 \cdot 5} = 20\ \%$.

By referring to Table XXIV., column 7, against 20 per cent. dividends, it will be seen that 11·9 years, or, say, 12 dividends, must be received. As the reserves have a life of two years only, the purchase at 30s. means paying for an assumption of ten years' life.

* See also p. 120.

TABLE XXIII.—Table of General Factors.

$r' = 3$ per cent. $r''' = 0$.

Year.	One Pound.		One Pound per Annum.		Sinking Fund.
n.	Amount. $(1 + r')^n$.	Present Value.* $\frac{1}{(1 + r)^n}$.†	Amount. $\frac{R^n - 1}{r'}$.	Present Value. $\frac{1}{r' + r''}$.	$r'' = \frac{r'}{R^n - 1}$.
(1)	(2)	(3)	(4)	(5)	(6)
1	1·03000	·97087	1·00000	·97087	1·000000
2	1·06090	·94260	2·03000	1·91347	·492611
3	1·09273	·91514	3·09090	2·82861	·323530
4	1·12551	·88849	4·18363	3·71710	·239027
5	1·15927	·86261	5·30914	4·57971	·188355
6	1·19405	·83748	6·46841	5·41719	·154598
7	1·22987	·81309	7·66246	6·23028	·130506
8	1·26677	·78941	8·89234	7·01969	·112456
9	1·30477	·76642	10·15911	7·78611	·098434
10	1·34392	·74409	11·46388	8·53020	·087231
11	1·38423	·72242	12·80780	9·25262	·078077
12	1·42576	·70138	14·19203	9·95400	·070462
13	1·46853	·68095	15·61779	10·63496	·064030
14	1·51259	·66112	17·08632	11·29607	·058526
15	1·55797	·64186	18·59891	11·93794	·053767
16	1·60471	·62317	20·15688	12·56110	·049611
17	1·65285	·60502	21·76159	13·16612	·045953
18	1·70243	·58739	23·41444	13·75351	·042709
19	1·75351	·57029	25·11687	14·32380	·039814
20	1·80611	·55368	26·87037	14·87748	·037216
21	1·86029	·53755	28·67649	15·41502	·034872
22	1·91610	·52189	30·53678	15·93692	·032747
23	1·97359	·50669	32·45288	16·44361	·030814
24	2·03279	·49193	34·42647	16·93554	·029047
25	2·09378	·47761	36·45926	17·41315	·027428
26	2·15659	·46369	38·55304	17·87684	·025938
27	2·22129	·45019	40·70963	18·32703	·024564
28	2·28794	·43708	42·93092	18·76411	·023293
29	2·35656	·42435	45·21885	19·18846	·022115
30	2·42726	·41199	47·57542	19·60044	·021019

† Delayed or deferred (d) years = $n - 1$ years.

* Present value $= \frac{1}{r' + r'' + r'''} = \frac{1}{(1 + r')^{d+1}} = \frac{1}{(1 + r')^n}$, as r'' or $\frac{r'}{R^n - 1}$ (the sinking fund) = 1.

† Note that d (the delay) begins after first year—*i.e.*, $d = n - 1$. See also Table XXII.

Basis of Table XXIV. (*continued*).

From Formula (53).
$$D = r' + r'' + r'''.$$
$$r'' = D - r' - r'''.$$

From Formula (54).
$$r'' = \frac{r'}{R^n - 1},$$

hence
$$R^n - 1 = \frac{r'}{D - r' - r'''},$$
$$n \log R = \log\left(\frac{r'}{D - r' - r'''} + 1\right).$$

Formula (55).
$$n = \frac{\log\left(1 + \frac{r'}{D - r' - r'''}\right)}{\log(1 + r')}.$$

TABLE XXIV.—Engineer's Table—Life in Terms of Dividends. Redeeming at 3 per cent.

1	2	3	4	5	6	7	8
Yearly Dividend in per cent. of Capital.	Redeeming Capital only. $r' = 0$ per cent. $r''' = 0$ "	Consols Basis. $r' = 3$ per cent. $r''' = 0$ "	Consols and 3 per cent. for Risks. $r' = 3$ per cent. $r''' = 3$ "	Consols and 5 per cent. for Risks. $r' = 3$ per cent. $r''' = 5$ "	Consols and 7 per cent. for Risks. $r' = 3$ per cent. $r''' = 7$ "	Consols and 10 p.c. for Risks $r' = 3$ per cent. $r''' = 10$ "	Consols and 15 p.c. for Risks. $r' = 3$ per cent. $r''' = 15$ "
	Years of Life (n).	Years of Life (n).	Years of Life (n).	Years of Life (n).	Years of Life (n).	Years of Life (n).	Years of Life (n).
3%	22·9	..	..	..	..	..	..
4%	18·9	46·9	..	..	..	..	..
5%	15·9	31·0	..	..	..	..	..
7%	11·9	18·9	46·9	..	..	..	..
10%	8·9	11·9	18·9	31·0	..	..	..
12%	7·5	9·5	13·9	18·9	31·0	..	..
15%	6·2	7·5	9·5	11·9	15·9	31·0	..
20%	4·7	5·6	6·6	7·5	8·9	11·9	31·0
25%	3·6	4·4	4·9	5·6	6·2	7·5	11·9
30%	3·2	3·5	4·0	4·4	4·7	5·6	7·5
35%	2·8	3·0	3·3	3·5	3·6	4·4	5·6
40%	2·4	2·6	2·8	3·0	3·2	3·6	4·4
45%	2·2	2·3	2·5	2·6	2·8	3·0	3·6
50%	1·9	2·1	2·2	2·3	2·4	2·6	3·0

For basis of table and formula, see p. 118.

Sensible Investor's Table (XXV.).

The query is often made as to the rate of dividends an investment must pay when one allows a given rate for

TABLE XXV.—Sensible Investor's Table.

Rate per cent. Investment must yield to be equivalent to Consols and pay r''' per cent. to cover Risk.

Risks of Loss. Redemption at 3 or 4 per cent.

n Years of Life.	1 Rate for Risk $r'''=3$ p. cent.		2 Rate for Risk $r'''=5$ p. cent.		3 Rate for Risk $r'''=7$ p. cent.		4 Rate for Risk $r'''=10$ p. cent.		5 Rate for Risk $r'''=15$ p. cent.		6 Rate for Risk $r'''=20$ p. cent.	
	$r'=3$ p. cent	$r'=4$ p. cent	$r'=3$ p. cent	$r'=4$ p. cent	$r'=3$ p. cent	$r'=4$ p. cent	$r'=3$ p. cent	$r'=4$ p. cent	$r'=3$ p. cent	$r'=4$ p. cent	$r'=3$ p. cent	$r'=4$ p. cent
1	..	..	..	..	..	..	..	..	..	..	..	..
2	55·3	55·0	57·3	57·0	59·3	59·0	62·3	62·0	67·3	67·0	72·3	72·0
3	38·3	38·0	40·3	40·0	42·3	42·0	45·3	45·0	50·3	50·0	55·3	55·0
4	29·9	29·5	31·9	31·5	33·9	33·5	38·9	36·5	43·9	41·5	48·9	46·5
5	24·8	24·5	26·8	26·5	28·8	28·5	31·8	31·5	36·8	36·5	41·8	41·5
6	21·5	21·1	23·5	23·0	25·5	25·0	28·5	28·0	33·5	33·0	38·5	38·0
7	19·0	18·7	21·0	20·7	23·0	22·7	26·0	25·7	31·0	30·7	36·0	35·7
8	17·2	16·8	19·2	18·8	21·2	20·8	24·2	23·8	29·2	28·8	34·2	33·8
9	15·8	15·4	17·8	17·4	19·8	19·4	22·8	22·4	27·8	27·4	32·8	32·4
10	14·7	14·3	16·7	16·3	18·7	18·3	21·7	21·3	26·7	26·3	31·7	31·3
11	13·8	13·4	15·8	15·4	17·8	17·4	20·8	20·4	25·8	25·4	30·8	30·4
12	13·0	12·6	15·0	14·6	17·0	16·6	20·0	19·6	25·0	24·6	30·0	29·6
13	12·4	12·0	14·4	14·0	16·4	16·0	19·4	19·0	24·4	24·0	29·4	29·0
14	11·8	11·5	13·8	13·5	15·8	15·5	18·8	18·5	23·8	23·5	28·8	28·5
15	11·4	11·0	13·4	13·0	15·4	15·0	18·4	18·0	23·4	23·0	28·4	28·0
16	11·0	10·6	13·0	12·6	15·0	14·6	18·0	17·6	23·0	22·6	28·0	27·6
17	10·6	10·2	12·6	12·2	14·6	14·2	17·6	17·2	22·6	22·2	27·6	27·2
18	10·3	9·9	12·3	11·9	14·3	13·9	17·3	16·9	22·3	21·9	27·3	26·9
19	10·0	9·6	12·0	11·6	14·0	13·6	17·0	16·6	22·0	21·6	27·0	26·6
20	9·7	9·4	11·7	11·4	13·7	13·4	16·7	16·4	21·7	21·4	26·7	26·4
25	8·7	8·4	10·7	10·4	12·7	12·4	15·7	15·4	20·7	20·4	25·7	25·4
30	8·1	7·8	10·1	9·8	12·1	11·8	15·1	14·8	20·1	19·8	25·1	24·8
35	7·6	7·4	9·6	9·4	11·6	11·4	14·6	14·4	19·6	19·4	24·6	24·4
40	7·3	7·0	9·3	9·0	11·3	11·0	14·3	14·0	19·3	19·0	24·3	24·0
45	7·0	6·8	9·0	8·8	11·0	10·8	14·0	13·8	19·0	18·8	24·0	23·8
50	6·9	6·6	8·9	8·6	10·9	10·6	13·9	13·6	18·9	18·6	23·9	23·6

risk, and the life of the property is known or assumed. As some may care to consider re-investing their interest at 4 per cent. per annum, a special column is allowed for this.

Example.—What rate of interest must a mine pay which has a life of 9 years, if an allowance of 5 per cent. be made for risk, with a redemption fund rate of 3 per cent.? In column 2 against 9 years of life will be found 17·8 per cent. as the rate of dividends one must expect of the mine.

Block Valuation, Bases of Tables Nos. XXVI. to XLII.

As discussed in the text, the two facts that in business life a greater rate is demanded of those undertakings understood to involve greater risk, and that, when dividends are deferred, this loss must be made good, are the basis of the valuation of blocks of ore, insomuch as each of these usually imply a different risk; also each block of one year's life means a difference in the deference factor applicable.

Table XXVI. is compiled on the supposition that, for ordinary calculation purposes, the mine is opened out into blocks, or may be considered to be made up of zones having a life of one year each. Evidently the tonnage in such a block or zone will depend upon the amount of ore mined each year—*i.e.*, the capacity of the reduction plant is the dominant factor.

Tables XXVI., XXVII., and XLII. are essentially the same as Table XXVI., but cover blocks with a greater life than one year.

As discussed in the Introduction, and as set out in Formula (15)—

The present value of each pound of annual profit in a block being C,

Formula (56). $$C = \frac{1}{(1+r'+r''')^d\left(r'+r'''+\frac{r'}{(1+r')^n-1}\right)},$$

where d is the years of deferrence, and n the years of life of the block.

When the block has one year's life only, the factor $\frac{r'}{(1+r)^n-1}$ becomes 1, hence Formula (56) may be written—

Formula (57). $$C = \frac{1}{(1+r'+r''')^{d+1}}$$

which is the basis of Table XXVI, where

d is the delay (in years) in exhausting the block ;

r' is 3 per cent., and

r''' the risk-rate from 0 to 20 heading the vertical columns.

This table is particularly interesting as showing the rapidly diminishing present values of the lower blocks of a mine with long life, more especially with the high risk-rates, which the lack of data concerning them necessarily implies.

TABLE XXVI.—Real or Present Value of Unit of Annual Profit in a Block of Ore.

For Block of 1 Year's Life only. Varying Risk-Rates (r''').

(d) The delay in years in exhausting the Block.	Risk-Rate $r''' = 0$ per cent.	Risk-Rate $r''' = 1$ per cent.	Risk-Rate $r''' = 2$ per cent.	Risk-Rate $r''' = 3$ per cent.	Risk-Rate $r''' = 4$ per cent.	Risk-Rate $r''' = 5$ per cent.	Risk-Rate $r''' = 6$ per cent.	Risk-Rate $r''' = 7$ per cent.	Risk-Rate $r''' = 8$ per cent.	Risk-Rate $r''' = 9$ per cent.	Risk-Rate $r''' = 10$ per cent.
0	·97087	·96154	·95238	·94340	·93458	·92593	·91743	·90909	.90090	·89286	·88496
1	·94260	·92456	·90703	·89000	·87344	·85734	·84168	·82645	·81162	·79719	·78315
2	·91514	·88900	·86384	·83962	·81630	·79383	·77218	·75131	·73119	·71178	·69305
3	·88849	·85480	·82270	·79209	·76290	·73503	·70843	·68301	·65873	·63552	·61332
4	·86261	·82193	·78353	·74726	·71299	·68058	·64993	·62092	·59345	·56743	·54276
5	·83748	·79031	·74622	·70496	·66634	·63017	·59627	·56447	·53464	·50663	·48032
6	·81309	·75992	·71068	·66506	·62275	·58349	·54703	·51316	·48166	·45235	·42506
7	·78941	·73069	·67684	·62741	·58201	·54027	·50187	·46651	·43392	·40388	·37616
8	·76642	·70259	·64461	·59190	·54393	·50025	·46043	·42410	·39092	·36061	·33288
9	·74409	·67556	·61391	·55839	·50835	·46319	·42241	·38554	·35218	·32197	·29459
10	·72242	·64958	·58468	·52679	·47509	·42888	·38753	·35049	·31728	·28748	·26070
11	·70138	·62460	·55684	·49697	·44401	·39711	·35553	·31863	·28584	·25667	·23070
12	·68095	·60057	·53032	·46884	·41496	·36770	·32618	·28966	·25751	·22917	·20416
13	·66112	·57748	·50507	·44230	·38782	·34046	·29925	·26333	·23200	·20462	·18068
14	·64186	·55526	·48102	·41727	·36245	·31524	·27454	·23939	·20900	·18269	·15989
15	·62317	·53391	·45811	·39365	·33873	·29189	·25187	·21763	·18829	·16312	·14150
16	·60502	·51337	·43630	·37136	·31657	·27027	·23107	·19784	·16963	·14564	·12522
17	·58739	·49363	·41552	·35034	·29586	·25025	·21199	·17986	·15282	·13004	·11081
18	·57029	·47464	·39573	·33051	·27651	·23171	·19449	·16351	·13768	·11611	·09806
19	·55368	·45639	·37689	·31180	·25842	·21455	·17843	·14864	·12403	·10366	·08680
20	·53755	·43883	·35894	·29416	·24151	·19866	·16370	·13513	·11174	·09256	·07680
21	·52189	·42196	·34185	·27751	·22571	·18394	·15018	·12285	·10066	·08264	·06796
22	·50669	·40573	·32557	·26180	·21095	·17032	·13778	·11168	·09069	·07379	·06014
23	·49193	·39012	·31007	·24698	·19715	·15770	·12640	·10153	·08170	·06582	.05322
24	·47761	·37512	·29530	·23300	·18425	·14602	·11597	·09230	·07361	·05882	·04710
25	·46369	·36069	·28124	·21981	·17220	·13520	·10639	·08391	·06631	·05252	·04168
26	·44019	·34682	·26785	·20737	·16093	·12519	·09761	·07628	·05974	·04690	·03689
27	·43708	·33348	·25509	·19563	·15040	·11591	·08955	·06934	·05382	·04187	·03264
28	·42435	·32065	·24295	·18456	·14056	·10733	·08215	·06304	·04849	·03738	·02889
29	·41199	·30832	·23138	·17411	·13137	·09938	·07537	·05731	·04368	·03338	·02556
30	·39999	·29646	·22036	·16425	·12277	·09202	·06915	·05210	·03935	·02980	·02262

For Basis of Table, see Formula (57).

TABLE XXVI.—*Continued.*

(d) The delay in years in exhausting the block.	Risk-Rate $r''' = 11$ per cent.	Risk-Rate $r''' = 12$ per cent.	Risk-Rate $r''' = 13$ per cent.	Risk-Rate $r''' = 14$ per cent.	Risk-Rate $r''' = 15$ per cent.	Risk-Rate $r''' = 16$ per cent.	Risk-Rate $r''' = 17$ per cent.	Risk-Rate $r''' = 18$ per cent.	Risk-Rate $r''' = 19$ per cent.	Risk-Rate $r''' = 20$ per cent.
0	·87719	·86956	·86207	·85470	·84746	·84034	·83333	·82645	·81967	·81301
1	·76947	·75614	·74316	·73051	·71818	·70616	·69444	·68301	·67186	·66098
2	·67497	·65752	·64066	·62437	·60863	·59341	·57870	·56447	·55071	·53738
3	·59208	·57175	·55229	·53365	·51579	·49867	·48225	·46651	·45140	·43690
4	·51937	·49718	·47611	·45611	·43711	·41905	·40188	·38554	·37000	·35520
5	·45559	·43233	·41044	·38984	·37043	·35214	·33490	·31863	·30328	·28878
6	·39964	·37594	·35383	·33319	·31392	·29592	·27908	·26333	·24859	·23478
7	·35056	·32690	·30502	·28478	·26604	·24867	·23257	·21763	·20377	·19088
8	·30751	·28426	·26295	·24340	·22546	·20897	·19381	·17986	·16702	·15519
9	·26974	·24718	·22668	·20804	·19106	·17560	·16150	·14864	·13690	·12617
10	·23662	·21494	·19542	·17781	·16192	·14756	·13459	·12284	·11221	·10257
11	·20756	·18691	·16846	·15197	·13722	·12400	·11216	·10152	·09198	·08339
12	·18207	·16253	·14523	·12989	·11629	·10420	·09346	·08390	·07539	·06780
13	·15971	·14133	·12519	·11102	·09855	·08757	·07789	·06934	·06180	·05512
14	·14010	·12289	·10793	·09489	·08352	·07359	·06490	·05731	·05065	·04481
15	·12289	·10686	·09304	·08110	·07078	·06184	·05409	·04736	·04152	·03643
16	·10780	·09292	·08020	·06932	·05998	·05196	·04507	·03914	·03403	·02962
17	·09456	·08080	·06914	·05924	·05083	·04367	·03756	·03235	·02789	·02408
18	·08295	·07026	·05961	·05064	·04308	·03669	·03130	·02673	·02286	·01958
19	·07276	·06110	·05138	·04328	·03650	·03084	·02608	·02209	·01874	·01592
20	·06383	·05313	·04430	·03699	·03094	·02591	·02174	·01826	·01536	·01294
21	·05599	·04620	·03819	·03162	·02622	·02177	·01811	·01509	·01259	·01052
22	·04911	·04017	·03292	·02702	·02220	·01830	·01509	·01247	·01032	·00855
23	·04308	·03493	·02838	·02310	·01883	·01538	·01258	·01031	·00846	·00695
24	·03780	·03038	·02446	·01974	·01596	·01292	·01048	·00852	·00693	·00565
25	·03315	·02641	·02109	·01687	·01352	·01086	·00873	·00704	·00568	·00460
26	·02908	·02297	·01818	·01442	·01146	·00912	·00728	·00582	·00466	·00374
27	·02551	·01997	·01567	·01232	·00971	·00767	·00607	·00481	·00382	·00304
28	·02237	·01737	·01351	·01053	·00823	·00644	·00505	·00397	·00313	·00247
29	·01963	·01510	·01165	·00900	·00697	·00541	·00421	·00328	·00256	·00201
30	·01721	·01313	·01004	·00769	·00591	·00455	·00351	·00271	·00021	·00163

TABLE XXVII.—r''' or Risk-rate = 2 per cent.

Real or Present Value of £1 or $1 of Annual Profit in Block.

Rate to be realised (s) = 5 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0*	·95238	1·84294	2·67716	3·45988	4·19543
1	·90703	1·75513	2·54966	3·29511	3·99568
2	·86384	1·67162	2·42825	3·13814	3·80546
3	·82273	1·59204	2·31262	2·98873	3·62424
4	·78353	1·51621	2·20250	2·84640	3·45173
5	·74622	1·44401	2·09762	2·71090	3·28731
6					
7					
8					
9					
10					

For basis of table and formula, see p. 123 and Appendix A.

* In using these tables it is important to note that "delay" means the time after the end of the first year; hence those entries against "0" correspond to the present value shown in some tables where deferrence is called 1. Accurate to four places.

TABLE XXVIII.—r''' OR RISK-RATE = 3 PER CENT.

REAL OR PRESENT VALUE OF £1 OR $1 OF ANNUAL PROFIT IN BLOCK

Rate to be realised (s) = 6 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
1	·94340	1·80959	2·62736	3·34415	4·02657
1	·89000	1·70715	2·47862	3·15487	3·79867
2	·83962	1·61054	2·33836	2·97628	3·58359
3	·79209	1·51936	2·20599	2·80781	3·38078
4	·74726	1·43336	2·08112	2·64887	3·18939
5	·70496	1·35223	1·96333	2·49894	3·00888
6					
7					
8					
9					
10					

For basis of table and formula, see p. 123.

TABLE XXIX.—r''' OR RISK-RATE = 4 PER CENT.

REAL OR PRESENT VALUE OF £1 OR $1 OF ANNUAL PROFIT IN BLOCK.

Rate to be realised (s) = 7 per cent.

Years of delay (d) in realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·93458	1·77743	2·54110	3·23596	3·87072
1	·87344	1·66116	2·37485	3·02425	3·61755
2	·81630	1·55248	2·21951	2·82638	3·38089
3	·76290	1·45092	2·07430	2·64152	3·15966
4	·71299	1·35601	1·93851	2·46871	2·95299
5	·66634	1·26728	1·81176	2·30718	2·75976
6					
7					
8					
9					
10					

TABLE XXX.—r''' or Risk-rate = 5 per cent.

Real or Present Value of £1 or $1 of Annual Profit in Block.

Rate to be realised (s) = 8 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·92593	1·74639	2·47813	3·13453	3·72641
1	·85734	1·61702	2·29456	2·90234	3·45038
2	·79383	1·49725	2·12460	2·68735	3·19480
3	·73503	1·38634	1·96722	2·48829	2·95815
4	·68058	1·28365	1·82150	2·30397	2·73902
5	·63017	1·18857	1·68656	2·13329	2·53620
6					
7					
8					
9					
10					

TABLE XXXI.—r''' or Risk-rate = 7 per cent.

Real or Present Value of £1 or $1 of Annual Profit in Block.

Rate to be realised (s) = 10 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·90909	1·68745	2·36111	2·94959	3·46796
1	·82645	1·53404	2·14646	2·68147	3·15268
2	·75132	1·39458	1·95133	2·43770	2·86608
3	·68301	1·26780	1·77393	2·21609	2·60552
4	·62092	1·15255	1·61268	2·01463	2·36866
5	·56447	1·04777	1·46606	1·83147	2·15336
6					
7					
8					
9					
10					

TABLE XXXII.—r''' or Risk-rate = 9 per cent.

Real or Present Value of £1 or $1 of Annual Profit in Block.

Rate to be realised (s) = 12 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·89286	1·63236	2·25464	2·78531	3·24302
1	·79719	1·45746	2·01317	2·48688	2·89555
2	·71178	1·30131	1·79738	2·22043	2·58532
3	·63552	1·16188	1·60481	1·98252	2·30832
4	·56743	1·03740	1·43286	1·77011	2·06020
5	·50663	·92624	1·04397	1·48034	1·84016
6					
7					
8					
9					
10					

TABLE XXXIII.—r''' or Risk-rate = 10 per cent.

Real or Present Value of £1 or $1 of Annual Profit in Block.

Rate to be realised (s) = 13 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·88496	1·60614	2·20493	2·70981	3·14120
1	·78315	1·42136	1·95126	2·39808	2·77978
2	·69305	1·25784	1·72679	2·12220	2·46002
3	·61332	1·11314	1·52812	1·87804	2·17699
4	·54276	·98508	1·35232	1·66196	1·92656
5	·48032	·87175	1·19674	1·47076	1·70491
6					
7					
8					
9					
10					

TABLE XXXIV.—r''' or Risk-rate = 12 per cent.

Real or Present Value of £1 or $1 of Annual Profit in Block.

Rate to be realised (s) = 15 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·86956	1·55615	2·11180	2·57052	2·95548
1	·75614	1·35318	1·83634	2·23523	2·56998
2	·65752	1·17667	1·59682	1·94368	2·23477
3	·57175	1·02319	1·38854	1·69016	1·94328
4	·49718	·88973	1·20743	1·46970	1·68980
5	·43233	·77369	1·04995	1·27800	1·46962
6	·37594	·67277	·91299	1·11129	1·27776
7	·32690	·58502	·79390	·96635	1·11107
8	·28426	·50871	·69035	·84030	·96616
9	·24719	·44236	·60031	·73070	·84009
10	·21494	·38466	·52201	·63539	·73057

TABLE XXXV.—r''' or Risk-rate = 15 per cent.

Real or Present Value of £1 or $1 of Annual Profit in Block.

Rate to be realised (s) = 18 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·84746	1·48674	1·98598	2·38648	2·71478
1	·71818	1·25995	1·68303	2·02244	2·30066
2	·60863	1·06776	1·42630	1·71393	1·94971
3	·51579	·90488	1·20873	1·45249	1·65230
4	·43711	·76685	1·02434	1·23092	1·40025
5	·37043	·64987	·86809	1·04314	1·18667
6	·31393	·55074	·73567	·88402	1·00565
7	·26604	·46673	·62345	·74918	·85225
8	·22546	·39553	·52835	·63489	·72225
9	·19107	·33520	·44775	·53804	·61207
10	·16191	·28407	·37945	·45597	·51871

TABLE XXXVI.—r''' OR RISK-RATE = 17 PER CENT.
REAL OR PRESENT VALUE OF £1 OR $1 OF ANNUAL PROFIT IN BLOCK.
Rate to be realised (s) 20 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0*	·83333	1·44381	1·91011	2·27776	2·57497
1	·69444	1·20318	1·59176	1·89814	2·14580
2	·57870	1·00265	1·32646	1·58178	1·78817
3	·48225	·83554	1·10539	1·31815	1·49014
4	·40188	·69628	·92116	1·09846	1·24179
5	·33490	·58024	·76763	·90537	1·03483
6	·27908	·48353	·63969	·76282	·86236
7	·23257	·40294	·53308	·63568	·71863
8	·19381	·33578	·44422	·52973	·59886
9	·16151	·27982	·37019	·44144	·49005
10	·13459	·23318	·30849	·36787	·41588

* Dividends end first year is no delay, being the ordinary occurrence.

TABLE XXXVII.—r''' OR RISK-RATE = 20 PER CENT.
REAL OR PRESENT VALUE OF £1 OR $1 OF ANNUAL PROFIT IN BLOCK.
Rate to be realised (s) = 23 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·81301	1·38387	1·80658	2·13206	2·39034
1	·66098	1·12509	1·46877	1·73337	1·94337
2	·53738	·91471	1·19441	1·40924	1·58000
3	·43690	·74367	·97083	1·14574	1·24454
4	·35520	·60471	·78929	·93149	1·04433
5	·28878	·49155	·64211	·75779	·84960
6	·23478	·39964	·52169	·61570	·69029
7	·19088	·32491	·42415	·50066	·56121
8	·15517	·26415	·34484	·40697	·45627
9	·12617	·21476	·28036	·33087	·37095
10	·10257	·17460	·22793	·26900	·30158

TABLE XXXVIII.—r''' or Risk-rate = 25 per cent.

Real or Present Value of £1 or $1 of Annual Profit in Block.

Rate realised (s) = 28 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·78125	1·29431	1·65692	1·92667	2·13516
1	·61035	1·01112	1·29446	1·50521	1·66808
2	·47684	·78999	1·01131	1·17594	1·30320
3	·37253	·61718	·79008	·91871	1·01812
4	·29104	·42817	·61725	·71774	·79514
5	·22737	·37610	·48223	·56074	·62145
6	·17764	·29429	·37674	·43808	·48548
7	·13878	·22992	·29433	·24225	·37928
8	·10842	·17962	·22994	·26738	·29631
9	·08470	·14033	·17964	·20889	·23149
10	·06617	·10963	·14035	·16320	·18085

TABLE XXXIX.—r''' or Risk-rate = 30 per cent.

Real or Present Value of £1 or $1 of Annual Profit in Block.

Rate realised (s) = 33 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·75188	1·21564	1·53015	1·75738	1·92920
1	·56532	·91402	1·15050	1·32133	1·45054
2	·42505	·68732	·86503	·99348	1·09062
3	·31959	·51719	·65039	·74698	·82001
4	·24029	·38851	·48902	·56164	·61655
5	·18067	·29212	·36768	·42229	·46357
6	·13584	·21963	·27645	·31751	·34855
7	·10214	·16514	·20786	·23873	·26207
8	·07680	·12416	·15629	·17947	·19704
9	·05774	·09336	·11751	·13496	·14815
10	·04341	·07010	·08835	·10147	·11139

TABLE XL.—r''' or Risk-rate = 35 per cent.

Real or Present Value of £1 or $1 of Annual Profit in Block.

Rate realised (s) = 38 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·72464	1·14598	1·42140	1·61543	1·75948
1	·52510	·83043	1·03000	1·17060	1·27499
2	·38051	·60176	·74638	·84826	·92390
3	·27573	·43606	·54085	·61468	·66963
4	·19980	·31598	·39192	·44542	·48514
5	·14479	·22895	·28400	·32277	·35155
6	·10492	·16592	·20580	·23389	·25480
7	·07603	·12023	·14913	·16949	·18460
8	·05509	·08713	·10806	·12282	·13377
9	·03992	·06313	·07831	·08900	·09693
10	·03002	·04575	·05674	·06449	·07024

TABLE XLI.—r''' or Risk-rate = 40 per cent.

Real or Present Value of £1 or $1 of Annual Profit in Block.

Rate to be realised (s) = 43 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·69930	1·08388	1·32701	1·49470	1·61721
1	·48902	·75796	·92803	1·04525	1·13091
2	·34197	·53004	·64898	·73094	·79085
3	·23914	·37066	·45383	·51115	·55304
4	·16723	·25920	·31736	·35719	·38674
5	·11694	·18126	·22193	·24996	·27045
6	·08178	·12675	·15520	·17480	·18912
7	·05719	·08864	·10853	·12224	·13225
8	·03999	·06199	·07589	·08548	·09249
9	·02797	·04335	·05307	·05978	·06468
10	·01956	·03031	·03711	·04180	·04523

TABLE XLII.—r''' or Risk-rate = 50 per cent.

Real or Present Value of £1 or \$1 of Annual Profit in Block.

Rate to be realised (s) = 53 per cent.

Years of Delay (d) in Realising Dividends from Block.	Life of Block (n) in Years.				
	1	2	3	4	5
0	·65359	·97789	1·17160	1·30034	1·38207
1	·42719	·63914	·76576	·84989	·90986
2	·27921	·41774	·50050	·55548	·59468
3	·18249	·27303	·32712	·36307	·38868
4	·11927	·17845	·21380	·23730	·25404
5	·07796	·11664	·13974	·15510	·16604
6	·05095	·07623	·09133	·10136	·10852
7	·03330	·04938	·05969	·06625	·07093
8	·02177	·03257	·03902	·04330	·04636
9	·01423	·02128	·02550	·02830	·03030
10	·00961	·01391	·01667	·01850	·01980

Tables of Real Values of Shares, Nos. XLIII. to LI.

Both engineers and investors have constant occasion to appraise the present or real value of shares, assuming, of course, that the data furnished regarding the amount, continuity, and number of dividends, is reliable.

As will be seen, each table is based on a different allowance for risk, such as the investor may care to allow, in accordance with his own information and views. The number of annual dividends or life is shown each year up to 20 years, and thereafter each five years up to 50 years.

The tables are calculated on the assumption that the dividends shall accumulate at 3 per cent. only, as in

accordance with the general idea set out elsewhere, a rate greater than 3 per cent. involves appreciable risk with which it were inaccurate to saddle the sinking fund if risks are to be seggregated, this being the basis of all the writer's calculations.

Example.—Shares are offered at 25s. paying 25 per cent. annually on the par value. The mine is given a life of 8 years, and the investors believe that the political conditions and management are such as to warrant one in allowing a risk-rate of 5 per cent. per annum, or a return of 8 per cent. Are the shares a good purchase at 25s.? The return being 25 per cent. at par means $\frac{\cdot 25}{1\cdot 25}$, or 20 per cent., on the market selling price. Looking up Table XLVII., which allows a risk-rate of 5 per cent., it is seen that under the column of annual dividends, at 20 per cent., and a life of 8 years, the real or present value is £1·03, or say 20s., hence the shares stand at too high a figure when selling at 25s.

Bases of Tables XLIII. and LI.—The discussion of Formulæ (1), (2), (3), and (4) in the Introduction will make clear the philosophy of the tables, they being simply the expression of

Formula (58). $$C = \frac{D}{r' + r'' + r'''}.$$

A separate table is allowed each risk-rate from 1 to 20 per cent., and the n or life of the property varies from 1 to 50 years.

TABLE XLIII.—Real Value of Shares, allowing 1 per cent. for Risk.

i.e., A Return of 4 per cent.

Years of Life (n).	$r' = 3$ per cent.					$r'' =$ variable.						$r''' = 1$ per cent.			
	Annual Dividends in percentage.—Capital at par.														
	3%	4%	5%	6%	7%	10%	12%	15%	20%	25%	30%	35%	40%	45%	50%
1	·03	·04	·05	·06	·07	·0962	·12	·14	·19	·24	·29	·34	·38	·43	·48
2	·06	·08	·09	·11	·13	·1878	·23	·28	·38	·47	·56	·66	·75	·85	·94
3	·08	·11	·14	·17	·19	·2751	·33	·41	·55	·69	·83	·96	1·10	1·24	1·38
4	·11	·14	·18	·21	·25	·3584	·43	·54	·72	·90	1·08	1·25	1·43	1·61	1·79
5	·13	·18	·22	·26	·31	·4379	·53	·66	·88	1·09	1·31	1·53	1·75	1·97	2·19
6	·15	·21	·26	·31	·36	·5139	·62	·77	1·03	1·28	1·54	1·80	2·06	2·31	2·57
7	·18	·23	·29	·35	·41	·5865	·70	·88	1·17	1·47	1·76	2·05	2·35	2·64	2·93
8	·20	·26	·33	·39	·46	·6559	·79	·98	1·31	1·64	1·97	2·30	2·62	2·95	3·28
9	·22	·29	·36	·43	·51	·7224	·87	1·08	1·44	1·81	2·17	2·53	2·89	3·25	3·61
10	·24	·31	·39	·47	·55	·7860	·94	1·18	1·57	1·96	2·36	2·75	3·14	3·54	3·93
11	·25	·34	·42	·51	·59	·8469	1·02	1·27	1·69	2·12	2·54	2·96	3·39	3·81	4·23
12	·27	·36	·45	·54	·63	·9053	1·09	1·36	1·81	2·26	2·72	3·17	3·62	4·07	4·53
13	·29	·38	·48	·58	·67	·9613	1·15	1·44	1·92	2·40	2·88	3·36	3·85	4·33	4·81
14	·30	·41	·51	·61	·71	1·0150	1·22	1·52	2·03	2·54	3·04	3·55	4·06	4·57	5·07
15	·32	·43	·53	·64	·75	1·0665	1·28	1·60	2·13	2·67	3·20	3·73	4·27	4·80	5·33
16	·33	·45	·56	·67	·78	1·1159	1·34	1·67	2·23	2·79	3·35	3·91	4·46	5·02	5·58
17	·35	·47	·58	·70	·81	1·1634	1·40	1·75	2·33	2·91	3·49	4·07	4·65	5·24	5·82
18	·36	·48	·60	·73	·85	1·2091	1·45	1·81	2·42	3·02	3·63	4·23	4·84	5·44	6·05
19	·38	·50	·63	·75	·88	1·2529	1·50	1·88	2·51	3·13	3·76	4·39	5·01	5·64	6·26
20	·39	·52	·65	·78	·91	1·2951	1·55	1·94	2·59	3·24	3·89	4·53	5·18	5·83	6·48
25	·44	·59	·74	·90	1·04	1·4831	1·78	2·22	2·97	3·71	4·45	5·19	5·93	6·67	7·42
30	·49	·66	·82	·98	1·15	1·6388	1·97	2·46	3·28	4·10	4·92	5·74	6·56	7·37	8·19
35	·53	·71	·88	1·06	1·24	1·7687	2·12	2·65	3·54	4·42	5·31	6·19	7·07	7·96	8·84
40	·56	·75	·94	1·13	1·31	1·8775	2·25	2·82	3·75	4·69	5·63	6·57	7·51	8·45	9·39
45	·59	·79	·98	1·18	1·38	1·9691	2·36	2·95	3·94	4·92	5·91	6·89	7·88	8·86	9·85
50	·61	·82	1·02	1·23	1·43	2·0464	2·46	3·07	4·09	5·12	6·14	7·16	8·19	9·21	10·23

TABLE XLIV.—Real Value of Shares, allowing 2 per cent. for Risk.

i.e., A Return of 5 per cent.

Years of Life (n).	$r' = 3$ per cent.						$r'' =$ variable.					$r''' = 2$ per cent.			
	Annual Dividends in per cent.—Capital at par.														
	3%	4%	5%	6%	7%	10%	12%	15%	20%	25%	30%	35%	40%	45%	50%
1	·03	·04	·05	·06	·07	·0952	·11	·14	·19	·24	·29	·33	·38	·43	·48
2	·06	·07	·09	·11	·13	·1843	·22	·28	·37	·46	·55	·64	·74	·83	·92
3	·08	·11	·13	·16	·19	·2677	·32	·40	·54	·67	·80	·94	1·07	1·20	1·34
4	·10	·14	·17	·21	·24	·3460	·42	·52	·69	·86	1·04	1·21	1·38	1·56	1·73
5	·13	·17	·21	·25	·29	·4195	·50	·63	·84	1·05	1·26	1·47	1·68	1·89	2·10
6	·15	·20	·24	·29	·34	·4888	·59	·73	·98	1·22	1·47	1·71	1·96	2·20	2·44
7	·17	·22	·28	·33	·39	·5540	·66	·83	1·11	1·38	1·66	1·94	2·22	2·49	2·77
8	·18	·25	·31	·37	·43	·6155	·74	·92	1·23	1·54	1·85	2·15	2·46	2·77	3·08
9	·20	·27	·34	·40	·47	·6737	·81	1·01	1·35	1·68	2·02	2·36	2·69	3·03	3·37
10	·22	·29	·36	·44	·51	·7287	·87	1·09	1·46	1·82	2·19	2·55	2·91	3·28	3·64
11	·23	·31	·39	·47	·55	·7808	·94	1·17	1·56	1·95	2·34	2·73	3·12	3·51	3·90
12	·25	·33	·42	·50	·58	·8301	1·00	1·25	1·66	2·08	2·49	2·91	3·32	3·74	4·15
13	·26	·35	·44	·53	·61	·8770	1·05	1·32	1·74	2·19	2·63	3·07	3·51	3·95	4·38
14	·28	·37	·46	·55	·64	·9314	1·11	1·38	1·84	2·30	2·76	3·22	3·69	4·15	4·61
15	·29	·39	·48	·58	·67	·9637	1·16	1·45	1·93	2·41	2·89	3·37	3·85	4·34	4·82
16	·30	·40	·50	·60	·70	1·0039	1·20	1·51	2·01	2·51	3·01	3·51	4·02	4·52	5·02
17	·31	·42	·52	·63	·73	1·0422	1·25	1·56	2·08	2·61	3·13	3·65	4·17	4·69	5·21
18	·32	·43	·54	·65	·76	1·0786	1·29	1·62	2·16	2·70	3·24	3·78	4·31	4·85	5·39
19	·33	·45	·56	·67	·78	1·1134	1·34	1·67	2·23	2·78	3·34	3·90	4·45	5·01	5·57
20	·34	·46	·57	·69	·80	1·1466	1·38	1·72	2·29	2·87	3·44	4·01	4·59	5·16	5·73
25	·39	·52	·65	·77	·90	1·2915	1·55	1·94	2·58	3·23	3·87	4·52	5·17	5·81	6·46
30	·42	·56	·70	·84	·99	1·4081	1·69	2·11	2·82	3·52	4·22	4·93	5·63	6·34	7·04
35	·45	·60	·75	·90	1·05	1·5029	1·80	2·25	3·01	3·76	4·51	5·26	6·01	6·76	7·51
40	·47	·63	·79	·95	1·11	1·5807	1·90	2·37	3·16	3·95	4·74	5·53	6·32	7·11	7·90
45	·49	·66	·82	·99	1·15	1·6451	1·97	2·47	3·29	4·11	4·94	5·76	6·58	7·40	8·23
50	·51	·68	·85	1·02	1·19	1·6988	2·04	2·55	3·40	4·25	5·10	5·95	6·80	7·64	8·49

TABLE XLV.—Real Value of Shares, allowing 3 per cent. for Risk.

i.e., A Return of 6 per cent. Redemption at 3 per cent.

Years of Life (n)	$r' = 3$ per cent.				$r'' =$ variable.					$r''' = 3$ per cent.				
	Annual Dividends in percentage.—Capital at par.													
	3%	4%	5%	7%	10%	12%	15%	20%	25%	30%	35%	40%	45%	50%
1	·03	·04	·05	·06	·09	·11	·14	·19	·23	·28	·33	·38	·42	·47
2	·05	·07	·09	·12	·18	·22	·27	·36	·45	·54	·63	·72	·81	·91
3	·08	·11	·13	·18	·26	·31	·39	·52	·65	·78	·91	1·04	1·17	1·30
4	·10	·13	·17	·23	·33	·40	·50	·67	·84	1·00	1·17	1·34	1·60	1·67
5	·12	·16	·20	·28	·40	·48	·60	·80	1·00	1·21	1·41	1·61	1·81	2·01
6	·14	·19	·23	·33	·46	·56	·70	·93	1·16	1·40	1·63	1·86	2·00	2·32
7	·16	·21	·26	·37	·52	·63	·79	1·05	1·31	1·57	1·84	2·10	2·36	2·62
8	·17	·23	·29	·40	·58	·69	·87	1·16	1·45	1·74	2·03	2·32	2·51	2·90
9	·19	·25	·31	·44	·63	·76	·95	1·26	1·56	1·89	2·21	2·52	2·84	3·15
10	·20	·27	·34	·47	·68	·82	1·02	1·36	1·70	2·04	2·38	2·72	3·06	3·40
11	·22	·30	·36	·51	·72	·87	1·09	1·44	1·81	2·17	2·52	2·90	3·26	3·62
12	·23	·31	·38	·54	·77	·92	1·15	1·53	1·92	2·30	2·68	3·07	3·45	3·83
13	·24	·32	·40	·56	·81	·97	1·21	1·61	2·01	2·42	2·82	3·23	3·63	4·04
14	·25	·34	·42	·59	·84	1·01	1·26	1·69	2·11	2·52	2·95	3·37	3·80	4·22
15	·26	·35	·44	·61	·88	1·05	1·31	1·77	2·20	2·63	3·08	3·51	3·95	4·40
16	·27	·36	·45	·64	·91	1·09	1·37	1·92	2·28	2·75	3·19	3·65	4·10	4·56
17	·28	·38	·47	·66	·94	1·13	1·41	1·89	2·36	2·83	3·31	3·77	4·25	4·71
18	·29	·39	·49	·68	·97	1·17	1·46	1·95	2·43	2·92	3·41	3·90	4·38	4·87
19	·30	·40	·50	·70	1·00	1·20	1·50	2·00	2·50	3·00	3·51	4·01	4·51	5·01
20	·31	·41	·51	·72	1·03	1·23	1·54	2·06	2·57	3·08	3·59	4·11	4·63	5·14
25	·34	·45	·55	·81	1·14	1·37	1·71	2·29	2·86	3·43	4·00	4·47	5·15	5·72
30	·37	·49	·61	·86	1·23	1·48	1·75	2·47	3·08	3·70	4·32	4·94	5·55	6·17
35	·39	·52	·65	·91	1·31	1·56	1·84	2·61	3·26	3·91	4·57	5·23	5·88	6·53
40	·41	·54	·68	·95	1·36	1·64	2·05	2·73	3·41	4·09	4·68	5·36	6·14	6·82
45	·42	·56	·70	·99	1·41	1·69	2·11	2·82	3·53	4·24	4·94	5·65	6·36	7·06
50	·44	·58	·72	1·02	1·45	1·74	2·20	2·90	3·55	4·36	5·08	5·81	6·53	7·25

For basis of Table and Formula, see p. 135.

TABLE XLVI.—Real Value of Shares, allowing 4 per cent. for Risk.

i.e., A Return of 7 per cent.

Years of Life (n)	$r' = 3$ per cent.					$r'' =$ variable.					$r''' = 4$ per cent.			
	Annual Dividends in percentage.—Capital at par.													
	3%	4%	5%	7%	10%	12%	15%	20%	25%	30%	35%	40%	45%	50%
1	·03	·04	·05	·06	·09	·11	·14	·18	·23	·28	·33	·37	·42	·46
2	·05	·07	·07	·12	·18	·22	·26	·36	·44	·53	·60	·72	·78	·88
3	·08	·10	·13	·18	·25	·30	·38	·50	·62	·76	·89	1·01	1·14	1·27
4	·10	·13	·16	·22	·32	·39	·49	·64	·80	·98	1·13	1·29	1·56	1·61
5	·12	·16	·19	·27	·39	·48	·58	·78	·96	1·17	1·35	1·56	1·74	1·93
6	·13	·18	·22	·31	·44	·54	·66	·88	1·12	1·32	1·55	1·78	2·00	2·23
7	·15	·20	·25	·35	·50	·60	·75	1·00	1·25	1·50	1·74	2·00	2·24	2·49
8	·16	·22	·27	·38	·55	·66	·82	1·10	1·37	1·65	1·92	2·20	2·47	2·74
9	·18	·24	·30	·41	·59	·72	·90	1·18	1·48	1·80	2·09	2·37	2·67	2·97
10	·19	·25	·32	·45	·64	·75	·95	1·28	1·59	1·92	2·23	2·56	2·76	3·18
11	·20	·27	·34	·47	·67	·81	1·01	1·34	1·69	2·02	2·36	2·70	3·04	3·38
12	·21	·28	·36	·50	·71	·84	1·06	1·42	1·78	2·13	2·49	2·85	3·20	3·56
13	·22	·30	·37	·53	·75	·90	1·12	1·50	1·87	2·25	2·63	3·00	3·40	3·75
14	·23	·31	·39	·55	·78	·93	1·16	1·58	1·95	2·34	2·73	3·12	3·51	3·90
15	·24	·32	·40	·57	·81	·96	1·21	1·62	2·02	2·43	2·83	3·24	3·63	4·04
16	·25	·33	·42	·58	·84	·99	1·25	1·64	2·09	2·52	2·92	3·28	3·76	4·18
17	·26	·34	·43	·60	·86	1·02	1·30	1·72	2·15	2·59	3·02	3·45	3·88	4·31
18	·27	·35	·44	·62	·89	1·05	1·33	1·78	2·22	2·67	3·10	3·56	4·00	4·44
19	·27	·36	·45	·64	·91	1·08	1·36	1·82	2·27	2·73	3·21	3·64	4·10	4·55
20	·28	·37	·47	·65	·93	1·11	1·40	1·86	2·33	2·80	2·26	3·73	4·20	4·66
25	·31	·41	·50	·70	·03	1·23	1·54	2·06	2·52	3·03	3·58	4·10	4·61	5·03
30	·33	·44	·55	·77	1·10	1·32	1·65	2·20	2·74	3·30	3·84	4·38	4·94	5·49
35	·35	·46	·58	·81	1·15	1·38	1·73	2·30	2·89	3·47	4·04	4·62	5·20	5·78
40	·36	·48	·60	·84	1·20	1·44	1·80	2·40	3·00	3·60	4·20	4·80	5·40	6·00
45	·37	·49	·62	·87	1·24	1·47	1·85	2·48	3·09	3·72	4·33	4·95	5·57	6·19
50	·38	·51	·63	·89	1·27	1·53	1·90	2·54	3·16	3·81	4·43	5·07	5·70	6·33

For basis of Table and Formula, see p. 135.

TABLE XLVII.—Real Value of Shares, allowing 5 per cent. for Risk.

i.e., A Return of 8 per cent.

Years of Life (n)	r' = 3 per cent.					r'' = variable.					r''' = 5 per cent.			
	Annual Dividends in percentage.—Capital at par.													
	3%	4%	5%	7%	10%	12%	15%	20%	25%	30%	35%	40%	45%	50%
1	·028	·04	·05	·07	·093	·12	·14	·19	·23	·29	·35	·37	·42	·46
2	·052	·07	·09	·12	·173	·28	·26	·35	·43	·52	·63	·70	·80	·87
3	·074	·10	·12	·17	·248	·30	·37	·49	·62	·74	·84	·98	1·08	1·24
4	·094	·11	·16	·22	·313	·33	·47	·63	·78	·94	1·12	1·06	1·44	1·56
5	·012	·15	·19	·26	·373	·45	·56	·75	·93	1·12	1·33	1·50	1·71	1·86
6	·027	·17	·21	·29	·426	·51	·63	·85	1·06	1·27	1·47	1·70	1·89	2·13
7	·042	·19	·24	·32	·475	·57	·71	·95	1·18	1·42	1·68	1·90	2·16	2·37
8	·055	·21	·26	·36	·519	·63	·78	1·03	1·30	1·55	1·82	2·06	2·34	2·59
9	·068	·22	·28	·39	·560	·66	·84	1·12	1·40	1·68	1·96	2·24	2·52	2·80
10	·079	·24	·30	·42	·598	·72	·90	1·19	1·50	1·79	2·10	2·38	2·70	2·99
11	·089	·25	·32	·45	·633	·75	·94	1·26	1·58	1·89	2·24	2·52	2·88	3·16
12	·099	·27	·33	·46	·665	·81	1·00	1·33	1·66	1·99	2·31	2·66	2·97	3·32
13	·207	·28	·35	·49	·694	·84	1·03	1·39	1·73	2·07	2·45	2·78	3·15	3·47
14	·217	·29	·36	·50	·722	·87	1·08	1·44	1·80	2·17	2·52	2·88	3·24	3·61
15	·224	·30	·37	·52	·748	·90	1·02	1·49	1·87	2·24	2·59	2·99	3·33	3·74
16	·231	·31	·38	·52	·771	·93	1·15	1·54	1·92	2·31	2·66	3·08	3·42	3·84
17	·238	·32	·40	·56	·794	·96	1·19	1·59	1·98	2·38	2·80	3·18	3·60	3·97
18	·244	·33	·41	·58	·815	·98	1·22	1·63	2·03	2·44	2·87	3·26	3·69	4·07
19	·250	·33	·42	·59	·835	·99	1·25	1·67	2·08	2·50	2·94	3·34	3·78	4·17
20	·256	·34	·43	·60	·853	1·02	1·28	1·71	2·13	2·56	3·01	3·42	3·87	4·26
25	·279	·37	·46	·64	·931	1·11	1·40	1·86	2·32	2·79	3·22	3·72	4·14	4·65
30	·297	·39	·49	·68	·990	1·17	1·48	1·98	2·47	2·97	3·43	3·96	4·41	4·95
35	·310	·41	·52	·73	1·036	1·23	1·55	2·07	2·58	3·10	3·64	4·14	4·68	5·16
40	·322	·43	·54	·75	1·072	1·29	1·61	2·14	2·68	3·22	3·78	4·28	4·86	5·36
45	·330	·44	·55	·77	1·101	1·32	1·65	2·20	2·75	3·30	3·85	4·40	4·95	5·50
50	·337	·45	·56	·78	1·125	1·35	1·68	2·25	2·82	3·37	3·92	4·50	5·04	5·62

For basis of Table and Formula, see p. 135.

TABLE XLVIII.—Real Value of Shares, allowing 7 per cent. for Risk.

i.e., A Return of 10 per cent.

Years of Life (n).	r' = 3 per cent.					r'' = variable.				r''' = 7 per cent.				
	Dividends in percentage.—Capital at par.													
	3%	4%	5%	7%	10%	12%	15%	20%	25%	30%	35%	40%	45%	50%
1	·03	·04	·05	·06	·091	·12	·14	·18	·22	·27	·32	·36	·41	·45
2	·05	·07	·08	·11	·169	·21	·24	·33	·42	·49	·56	·66	·75	·84
3	·07	·09	·12	·17	·236	·27	·35	·47	·59	·70	·84	·94	1·08	1·18
4	·09	·12	·15	·21	·295	·36	·44	·59	·73	·88	1·05	1·18	1·35	1·47
5	·10	·14	·17	·24	·347	·42	·52	·69	·81	1·04	1·19	1·38	1·53	1·73
6	·12	·16	·20	·28	·393	·48	·59	·79	·88	1·18	1·40	1·58	1·80	1·96
7	·13	·17	·22	·31	·434	·51	·65	·87	1·08	1·30	1·54	1·74	1·98	2·17
8	·14	·19	·23	·32	·471	·57	·70	·94	1·17	1·41	1·61	1·88	2·07	2·35
9	·15	·20	·25	·35	·504	·60	·75	1·01	1·25	1·51	1·75	2·02	2·25	2·52
10	·16	·21	·27	·38	·534	·63	·80	1·07	1·33	1·60	1·89	2·14	2·43	2·67
11	·17	·22	·28	·39	·561	·66	·84	1·12	1·40	1·68	1·96	2·24	2·52	2·80
12	·18	·23	·29	·41	·587	·69	·88	1·17	1·46	1·76	2·03	2·34	2·61	2·93
13	·18	·24	·30	·42	·610	·72	·91	1·22	1·52	1·83	2·10	2·44	2·70	3·05
14	·19	·25	·31	·43	·631	·75	·94	1·26	1·57	1·89	2·17	2·52	2·79	3·15
15	·19	·26	·32	·45	·650	·76	·97	1·30	1·62	1·95	2·24	2·60	2·88	3·25
16	·20	·26	·33	·46	·668	·78	1·00	1·33	1·67	2·00	2·31	2·66	2·97	3·34
17	·20	·27	·34	·48	·685	·81	1·02	1·36	1·71	2·04	2·38	2·72	3·06	3·42
18	·21	·28	·35	·49	·701	·84	1·05	1·40	1·75	2·10	2·45	2·80	3·15	3·50
19	·21	·29	·36	·50	·715	·86	1·07	1·43	1·78	2·14	2·52	2·86	3·20	3·57
20	·22	·29	·36	·52	·729	·87	1·09	1·45	1·82	2·18	2·62	2·90	3·27	3·64
25	·23	·30	·39	·55	·785	·90	1·17	1·57	1·86	2·35	2·73	3·02	3·52	3·92
30	·25	·33	·41	·57	·826	·99	1·23	1·67	2·06	2·47	2·87	3·34	3·71	4·13
35	·26	·34	·43	·60	·858	1·02	1·28	1·71	2·14	2·57	3·01	3·42	3·86	4·29
40	·27	·35	·44	·62	·883	1·05	1·32	1·77	2·21	2·65	3·08	3·54	3·97	4·41
45	·27	·36	·45	·63	·903	1·08	1·35	1·80	2·25	2·70	3·15	3·60	4·06	4·51
50	·28	·37	·46	·64	·918	1·11	1·37	1·83	2·30	2·75	3·22	3·66	4·13	4·59

For basis of Table and Formula, see p. 135.

TABLE XLIX.—Real Value of Shares, allowing 10 per cent. for Risk.

i.e., A Return of 13 per cent.

Years of Life (n)	r' = 3 per cent.					r'' = variable.				r''' = 10 per cent.				
	Dividends in percentage.—Capital at par.													
	3%	4%	5%	7%	10%	12%	15%	20%	25%	30%	35%	40%	45%	50%
1	·03	·03	·04	·06	·088	·10	·13	·17	·22	·26	·31	·34	·39	·44
2	·05	·06	·08	·11	·161	·19	·24	·32	·40	·48	·56	·62	·72	·80
3	·07	·09	·11	·15	·220	·27	·33	·44	·55	·66	·77	·88	·99	1·10
4	·08	·11	·13	·19	·271	·33	·40	·54	·62	·81	·94	1·08	1·21	1·35
5	·09	·13	·16	·22	·314	·39	·47	·63	·78	·94	1·10	1·26	1·44	1·57
6	·10	·14	·17	·24	·351	·42	·52	·70	·87	1·05	1·22	1·40	1·57	1·75
7	·11	·15	·19	·27	·384	·45	·57	·77	·96	1·15	1·34	1·54	1·62	1·92
8	·12	·16	·21	·29	·412	·48	·62	·82	1·03	1·24	1·44	1·64	1·82	2·06
9	·13	·17	·22	·31	·438	·51	·65	·87	1·08	1·31	1·54	1·74	1·98	2·19
10	·14	·18	·23	·32	·460	·54	·69	·92	1·15	1·38	1·61	1·84	2·07	2·30
11	·14	·19	·24	·33	·480	·57	·72	·96	1·20	1·44	1·68	1·92	2·16	2·40
12	·15	·20	·25	·35	·499	·60	·73	·99	1·24	1·47	1·75	1·98	2·25	2·49
13	·15	·21	·26	·36	·515	·62	·77	1·03	1·28	1·54	1·80	2·06	2·31	2·57
14	·16	·21	·26	·37	·530	·64	·80	1·06	1·32	1·59	1·85	2·12	2·38	2·65
15	·16	·22	·27	·38	·544	·66	·82	1·09	1·36	1·63	1·90	2·18	2·44	2·72
16	·17	·22	·28	·39	·557	·67	·84	1·11	1·39	1·67	1·94	2·22	2·49	2·78
17	·17	·23	·28	·40	·568	·68	·85	1·13	1·42	1·70	1·98	2·26	2·55	2·84
18	·17	·23	·29	·40	·579	·69	·86	1·15	1·44	1·73	2·02	2·30	2·60	2·89
19	·18	·23	·29	·41	·589	·70	·88	1·17	1·47	1·76	2·06	2·34	2·64	2·94
20	·18	·24	·30	·42	·598	·72	·90	1·19	1·50	1·79	2·09	2·38	2·69	2·99
25	·19	·25	·32	·44	·635	·75	·95	1·27	1·58	1·90	2·21	2·56	2·85	3·17
30	·20	·26	·33	·46	·662	·78	1·00	1·32	1·65	1·99	2·32	2·64	2·98	3·31
35	·20	·27	·34	·48	·682	·81	1·02	1·36	1·70	2·05	2·39	2·72	3·07	3·41
40	·21	·28	·35	·49	·698	·83	1·04	1·39	1·75	2·09	2·45	2·78	3·14	3·49
45	·21	·28	·35	·50	·710	·84	1·06	1·42	1·77	2·13	2·48	2·84	3·19	3·55
50	·22	·29	·36	·51	·720	·87	1·08	1·44	1·80	2·16	2·52	2·88	3·24	3·60

For basis of Table and Formula, see p. 135.

TABLE L.—Real Value of Shares, allowing 15 per cent. for Risk.

i.e., A Return of 18 per cent.

$r' = 3$ per cent. $r'' =$ variable. $r''' = 15$ per cent.

Dividends in percentage.—Capital at par.

Years of Life (n)	3%	4%	5%	7%	10%	12%	15%	20%	25%	30%	35%	40%	45%	50%
1	·02	·03	·04	·06	·085	·10	·12	·17	·21	·25	·28	·34	·36	·42
2	·04	·06	·07	·10	·149	·17	·22	·29	·37	·44	·49	·58	·63	·74
3	·06	·08	·10	·14	·198	·23	·30	·39	·49	·59	·70	·78	·90	·99
4	·07	·09	·12	·17	·239	·29	·36	·47	·59	·73	·84	·96	1·08	1·19
5	·08	·11	·13	·18	·271	·32	·40	·54	·67	·81	·91	1·08	1·17	1·35
6	·09	·12	·15	·21	·299	·35	·45	·60	·74	·89	1·05	1·20	1·35	1·49
7	·10	·13	·16	·22	·322	·38	·48	·64	·80	·96	1·12	1·28	1·44	1·61
8	·10	·14	·17	·24	·342	·41	·51	·68	·85	1·03	1·19	1·36	1·53	1·71
9	·11	·14	·18	·25	·359	·43	·53	·71	·89	1·07	1·26	1·42	1·62	1·79
10	·11	·15	·19	·26	·374	·45	·56	·75	·93	1·12	1·30	1·50	1·71	1·87
11	·12	·15	·19	·27	·387	·46	·58	·77	·96	1·16	1·35	1·54	1·75	1·93
12	·12	·16	·20	·28	·399	·47	·59	·79	·98	1·19	1·40	1·58	1·80	1·99
13	·12	·16	·20	·29	·410	·49	·61	·82	1·02	1·23	1·44	1·64	1·85	2·05
14	·12	·17	·21	·29	·419	·50	·62	·83	1·04	1·25	1·47	1·66	1·89	2·09
15	·13	·17	·21	·30	·428	·51	·64	·85	1·07	1·28	1·50	1·70	1·94	2·14
16	·13	·17	·22	·31	·435	·52	·65	·87	1·09	1·30	1·54	1·74	1·98	2·17
17	·13	·18	·22	·32	·442	·53	·67	·88	1·10	1·33	1·58	1·76	1·99	2·21
18	·13	·18	·22	·32	·449	·54	·67	·89	1·12	1·34	1·60	1·78	2·03	2·24
19	·14	·18	·23	·32	·455	·54	·68	·91	1·13	1·36	1·61	1·82	2·07	2·27
20	·14	·18	·23	·32	·460	·55	·69	·92	1·15	1·38	1·62	1·84	2·09	2·30
25	·15	·19	·24	·33	·482	·58	·72	·96	1·20	1·45	1·68	1·92	2·16	2·41
30	·15	·20	·25	·35	·497	·59	·74	·99	1·24	1·49	1·75	1·98	2·25	2·48
35	·15	·20	·25	·36	·509	·61	·76	1·02	1·27	1·53	1·78	2·04	2·30	2·54
40	·15	·21	·26	·36	·517	·62	·77	1·03	1·29	1·55	1·80	2·06	2·31	2·58
45	·16	·21	·26	·36	·524	·63	·78	1·05	1·31	1·57	1·81	2·10	2·32	2·62
50	·16	·21	·26	·36	·529	·63	·79	1·06	1·32	1·58	1·82	2·12	2·34	2·64

For basis of Table and Formula, see p. 135.

TABLE LI.—Real Value of Shares, allowing 20 per cent. for Risk.

i.e., A Return of 23 per cent.

Years of Life (n).	$r' = 3$ per cent.				r'' = variable.					$r''' = 20$ per cent.				
	Dividends in percentage.—Capital at par.													
	3%	4%	5%	7%	10%	12%	15%	20%	25%	30%	35%	40%	45%	50%
1	·02	·03	·04	·06	·081	·10	·12	·16	·20	·24	·28	·32	·36	·40
2	·04	·05	·07	·08	·138	·16	·20	·27	·35	·41	·49	·54	·63	·69
3	·05	·07	·09	·12	·181	·20	·27	·36	·45	·54	·63	·72	·81	·90
4	·06	·08	·11	·15	·213	·24	·32	·43	·53	·64	·77	·86	·99	1·06
5	·07	·09	·12	·17	·239	·28	·35	·48	·60	·71	·84	·96	1·08	1·19
6	·08	·10	·13	·18	·260	·32	·39	·52	·65	·78	·91	1·04	1·14	1·30
7	·08	·11	·14	·19	·277	·33	·41	·55	·69	·83	·98	1·10	1·26	1·38
8	·09	·12	·15	·21	·292	·35	·44	·58	·73	·88	1·05	1·16	1·34	1·46
9	·09	·12	·15	·22	·304	·36	·45	·61	·76	·91	1·08	1·22	1·35	1·52
10	·09	·13	·16	·22	·315	·37	·47	·63	·78	·94	1·10	1·26	1·44	1·57
11	·10	·13	·16	·22	·324	·38	·48	·65	·81	·97	1·12	1·30	1·46	1·62
12	·10	·13	·17	·23	·333	·40	·50	·67	·83	1·00	1·17	1·34	1·52	1·66
13	·10	·14	·17	·23	·340	·41	·51	·68	·85	1·02	1·18	1·36	1·53	1·70
14	·10	·14	·17	·24	·346	·42	·52	·69	·86	1·04	1·19	1·38	1·54	1·73
15	·11	·14	·18	·25	·352	·42	·53	·70	·88	1·06	1·23	1·40	1·56	1·76
16	·11	·14	·18	·25	·358	·43	·53	·71	·89	1·07	1·24	1·42	1·58	1·79
17	·11	·14	·18	·25	·362	·44	·54	·72	·91	1·09	1·25	1·44	1·60	1·81
18	·11	·15	·18	·25	·366	·44	·55	·73	·92	1·10	1·26	1·46	1·61	1·83
19	·11	·15	·18	·25	·370	·44	·55	·74	·93	1·11	1·27	1·48	1·62	1·85
20	·11	·15	·19	·26	·374	·45	·56	·74	·94	1·12	1·32	1·48	1·66	1·87
25	·12	·16	·19	·27	·389	·46	·58	·78	·97	1·17	1·33	1·56	1·71	1·94
30	·12	·16	·20	·28	·398	·47	·59	·80	·98	1·19	1·39	1·60	1·76	1·99
35	·12	·16	·20	·28	·406	·48	·61	·81	1·02	1·22	1·40	1·62	1·78	2·03
40	·12	·16	·20	·28	·411	·49	·61	·82	1·03	1·23	1·41	1·64	1·80	2·05
45	·12	·17	·21	·29	·415	·50	·62	·83	1·04	1·24	1·46	1·66	1·85	2·07
50	·12	·17	·21	·30	·419	·50	·62	·84	1·05	1·25	1·47	1·68	1·89	2·09

For basis of Table and Formula, see p. 135.

APPENDIX A.

ALLOWANCES FOR DELAYED DIVIDENDS.

In order that the underlying bases of delayed dividends be understood, it would seem necessary to go back to the very beginning of human industry, and we may say :—

(*a*) From the results of sowing and reaping, experience justifies the expectation of a return of the seed capital and a further increment ; this last corresponds to interest in commercial life with the transaction closed in one season. Where sowing produces several crops, as, say, from an orchard, the transaction is extended over several seasons.

(*b*) From the above return a part is put aside as a reserve against the next sowing, while a portion is used for sustenance. The first may be regarded as the prototype of the capital redemption fund, and the second as ordinary interest.

(*c*) Experience shows that crops fail, occasionally several seasons in succession, demonstrating the necessity of providing a reserve greater than that called for by the next sowing or planting. This experience of failure and provision to meet it first taught the lesson of deferrence of increment, risk, probability, and the formation

of a reserve fund, and that the magnitude of the latter should be a function of the number of untoward events.

(*d*) As commerce developed, it became increasingly a part of general experience that loss was a function of the data available, varying inversely, as, for instance, the greater the knowledge of routes and conditions, human and other, the fewer the failures. Experience also taught that the longer the trip or voyage undertaken the greater the probability of failure ; * but the length of the voyage being a function of the time, the habit developed of regarding the probability of failure as a function of the time as well as of specific data.

From the foregoing it is seen that probability, a series of ventures, risk a function of the time, risk a function of the data, interest, a reserve fund, a function of the number of failures (risk) and delay or deferrence, are all basic concepts, the outcome of bitter experience dating from the dawn of commerce—*i.e.*, are not mere academic postulates.

The great impediment to sound reasoning seems to be the confusion of hope with calculation. Assuredly hope of success impels to action, but it is or should be conditioned by conscious or unconscious rationation involving experience—*i.e.*, probability. Unless hope be moderated by such considerations, it becomes but a pleasurable mental excitement, as seen in most forms of gambling.

While all business is of necessity speculation or hazard, the calculation of risk largely distinguishes it from

* Note that for " number of failures " we now use " probability of failure—*i.e.*, probability postulates a number of events.

gambling. Speculation may not be definitely differentiated from ordinary business, but the magnitude of the reserve fund is, from the foregoing, greater in the case of speculation ; but the annual risk-rate is only an expression of this reserve, hence the more speculative an undertaking the greater the risk-rate called for.

The status of the engineer being that of an intermediary, or interpreter between the scientist and financier, it is imperative that his education should be sufficiently comprehensive to familiarise him with the discoveries of the one and the demands of the other ; assuming, of course, that the latter are consonant with economics in its broader aspects—*i.e.*, with civic proportion—a sense of which it is seen must find place even in calculations of profit.

In this office of interpreter the engineer is the one who legitimately sets the risk-rate by the light of his knowledge of science, and this position is one which calls for the analysis and comparison of the merits of different ventures, essentially expressed in monetary units.*

Perhaps the greatest difficulty the engineer has to meet is his necessity of facing the civic demand that an equivalent return be made for service rendered, or outlay made. For instance, the client investing in the more ordinary undertaking has two possible sources of profit in his mind ; that of realising upon the ignorance of others as expressed in the share market quoting inflated values, and that of profit from dividends received. While the share market is manifestly an indispensable adjunct to

* Note that "a fine showing," "favourable geological conditions," etc., are meaningless terms if not expressed in monetary units.

civilisation as a convenience in readily realising upon capital, yet the hope of the client to secure from it the windfalls of credulity or greed can find no legitimate place in the engineer's calculations.

From the above reasoning it will be evident that the generally accepted premises forming the basis of appraisal of business venture may be stated as follows :—

Premise I.—Capital outlay in a venture of a terminable life assumes the return of the original capital (reproduction rate, r'') ; an equal annual rate of contribution to the reserve fund (r''') * and a sustenance rate (r').

Corollary to Premise I.—The reserve fund contribution (r''') is an integral part of the annual increment, and represents both or either capital and the sustenance rate, when failure occurs to one or more in a series of ventures.

Premise II.—When dividends are delayed or deferred beyond one year from the date of investment, the losses due to the delay shall be made good by the subsequent payments.

Premise III.—A comparison of ventures entails the consideration of equal times of investment.

Premise IV.—The minimum rate of interest (r') is assumed to be the return from the investment representing the minimum risk.

* It is especially necessary to realise that the sustenance rate (r', or say 3 per cent.) is as vital as the return of capital itself ; further, that the accumulated reserve is really nothing but capital (and perhaps sustenance) when failure occurs in one or more of a series of investments.

The sustenance rate (r') is also taken as that of the security involving the minimum risk.

Premise V.—Only one rate for risk and one rate of redemption to be used when comparing two ventures.

Premise VI. — The sustenance and redemption rates shall be equal and represent the minimum rate (r').

In the following, two enterprises are considered, one of which shall yield (n) dividends commencing one year from date of investment, and the other be delayed (d) years beyond the other, but then to yield the same number (n) of dividends, and at the same rate.

Taking investment A, which yields equal and yearly dividends commencing one year from date of investment, it will be seen that for n years the return is $r' + r'' + r'''$, where r' is the sustenance rate, r'' the rate put aside and compounded at r' for n years when it equals the original capital; and r''', the stipulated annual contribution to the reserve fund. As at the end of n years investment A has not completed its term, according to premise III. it is necessary to re-invest the proceeds, which is made in a venture entailing no risk, hence calls for no risk-rate, nor r'' as it sells at par.

At the end of the first year investment A yields D per cent., or $(r' + r'' + r''')$ per cent., hence B makes its annual contribution to sustenance and the reserve whereby to protect the original capital. Investment B at the end of the first year yields nothing, and no contribution is made to the reserve fund. Similar results are had at the end of the second year, investment A

contributing to the reserve fund and B failing; and on for d years. At the end of the first year the contribution to the reserve and redemption fund made by A is invested in a venture entailing no risk, hence its capital is protected to that extent in case of failure to realise dividends later on. Investment B, on the other hand, has no such protection. The one way to secure protection to the latter, and to provide for the sustenance rate (r') is to find the necessary additional capital each year during the delay. The new capital, $(r' + r''') \frac{R^d - 1}{r'}$, will amount to $(r' - r''') \frac{R^d - 1}{r'} \frac{R^n - 1}{r'}$ in n years from the end of the deferred period. As seen by Formula (4), the present value of a deferred annuity or delayed dividend is—

Formula (59). $$C = \frac{1}{r' + r'' + r''' + r''''},$$

where r'''' is the annual rate or portion of the dividend (when received) to meet the loss due to the delay. This annual rate r'''' when invested at r' amounts in n years to $r'''' \frac{R^n - 1}{r'}$, which, as above, must meet the losses to the new capital and r' interest on same for n years—*i.e.*,

Formula (60).

$$r'''' \frac{R^n - 1}{r'} = (r' + r''') \frac{R^d - 1}{r'} + (r' + r''') \frac{R^d - 1}{r'} (r') \frac{R^n - 1}{r'}.$$

$$r'''' = \left[(r' + r''') \frac{R^d - 1}{r} \right] \left[\frac{r'}{R^n - 1} + r' \right],$$

but by Formula (13) $\dfrac{r'}{R^n - 1} = r''$, hence the present value—

Formula (61). $C = \dfrac{1}{r' + r'' + r''' + \left[(r' + r''')\dfrac{R^d - 1}{r'}(r' + r'')\right]}$.

This gives results identical with Mr. O'Donahue's * formula, which is—

Formula (62). $C = \dfrac{1}{r'' + r''(r' + r''')\dfrac{R^{n-d} - 1}{r'}}$.

But while the original capital has been protected during the period of delay by finding new capital to contribute to the reserve fund, how about an insurance for this new capital? Clearly none exists; it must look to the enterprise itself for repayment, hence calls for a corresponding risk-rate.

This new capital then at the end of the first year is $(r' + r''')$, and as it is also entitled to sustenance and an insurance rate, or $(r' + r''')$, it becomes $\dfrac{s\,(S^d - 1)}{s}$ at the end of the period of the delay; $s = (r' + r''')$ and $S = (1 + s)$.

From the reasoning preceding Formula (16)—

Formula (63).

$$C = \frac{1}{S^d\,(r'' + s)} = \frac{1}{(1 + r' + r''')^d\left(r' + r''' + \dfrac{r'}{R^n - 1}\right)}.$$

The essential point of difference between Formulæ (63) and (62), that of Mr. O'Donahue, is the failure on the part of the latter to recognise that the new capital

* See *The Valuation of Mineral Properties*, by T. A. O'Donahue.

is also entitled to the risk-rate, though it demands an insurance for the original capital. In a word, most writers on the subject, save the late Mr. Hoskold,* would appear to have overlooked the essence of a risk-rate, which is the identity of the reserve fund with capital itself when the broader aspects of insurance are considered.

It may be urged that as the capital remaining in the undertaking—*i.e.*, unredeemed by r''—is ever growing less, the risk-rate (r''') should be applied to the remaining part only. In reality one is bound by the conventions of the business world which does not fix a reserve or even a sinking fund, but merely exacts an annual rate which must satisfy the above requirements. In fact, selling at par is the ordinary sinking fund, but in order to intelligently compare the merit of different ventures, particularly those with a limited life, the above postulates have to be made. The student would do well to remember, however, that in order to serve the main purpose—*i.e.*, reduce the "personal equation"—in other words, to effectively eliminate "inspiration methods"—theories of valuation must conform to business concepts in order to find acceptance, which last is evidently the real aim.

This practical demand for a rate sufficient to meet the $r' + r'' + r'''$ requirement is productive of some apparent anomalies, as, for instance, the fact that r''' is made to accumulate to a larger sum in the case of longer

* While the writer's formula is developed differently from that of the late Mr. Hoskold, it is identical in construction, nor is there occasion to believe that the latter failed to overlook the underlying philosophy, even though unstated in his book, *Mine Valuers' Assistant.*

deferrence, even though the *rate for risk per annum* be made the same. This aggregate allowance for risk, or accumulation of reserve, is seen to be in accordance with experience, if one but contemplate the future which shows that uncertainty varies with the time.

In the case of two blocks of ore, each with the same risk-rate above the standard rate, but worked at different times, it will be seen—if one conceives of laying out new capital to meet the stipulation for contribution to the reserve during delay—that a portion of the risk-rate varies with the data, while the other, the standard portion, is fixed, though both demand an insurance fund varying with the time. At first thought, it would seem unusual to so penalise what purports to be an allowance for a difference in data only. In reality the premise is an annual rate which is made a function of the data, *and not a reserve fund* which shall vary with the data and remain independent of the time. In other words, the course followed is like that of the ordinary business world, the reserves of which are expressed by the rates of dividend alone, or the equivalent, their present value.

When a banker discounts two bills at 4 per cent., due at different times, the extra per cent. above, say, 3 per cent. may be said to represent an annual allowance for a shortage in data concerning the undertakings ; the ultimate contribution of each to the bank's reserve fund will vary, however, with the time, even though the data as expressed by the extra per cent. be the same.

It may be well to again point out that allowances for risk, whether for the mine as a whole, for the standard block, or for that addition to the risk rate designed to

allow for defective data, are much like the allowances for ignorance of certain facts when calculating the strength of beams. The latter calculations are of necessity wanting, to meet which we employ a factor of safety of hundreds of per cent. *Yet are these allowances sound, when they meet general acceptance, for the essence of accuracy, is the elimination so far as possible of the personal error, whether this be accidental or intentional; and standardised calculations even where defective serve this end.*

Angle.	Sine.	Tangent.	Co-tangent.	Cosine.	Angle.
0°	0	0	∞	1	90°
1	·0175	·0175	57·2900	·9998	89
2	·0349	·0349	28·6363	·9994	88
3	·0523	·0524	19·0811	·9986	87
4	·0698	·0699	14·3006	·9976	86
5	·0872	·0875	1·4301	·9962	85
6	·1045	·1051	9·5144	·9945	84
7	·1219	·1228	8·1443	·9925	83
8	·1392	·1405	7·1154	·9903	82
9	·1564	·1584	6·3138	·9877	81
10	·1736	·1763	5·6713	·9848	80
11	·1908	·1944	5·1446	·9816	79
12	·2079	·2126	4·7046	·9781	78
13	·2250	·2309	4·3315	·9744	77
14	·2419	·2493	4·0108	·9703	76
15	·2588	·2679	3·7321	·9659	75
16	·2756	2867	3·4874	·9613	74
17	·2924	·3057	3·2709	·9563	73
18	·3090	·3249	3·0777	·9511	72
19	·3256	·3443	2·9042	·9455	71
20	·3420	·3640	2·7475	·9397	70
21	·3584	·3839	2·6051	·9336	69
22	·3746	·4040	2·4751	·9272	68
23	·3907	·4245	2·3559	·9205	67
24	·4067	·4452	2·2460	·9135	66
25	·4226	·4663	2·1445	·9063	65
26	·4384	·4877	2·0503	·8988	64
27	·4540	·5095	1·9626	·8910	63
28	·4695	·5317	1·8807	·8830	62
29	·4848	·5543	1·8040	·8746	61
30	·5000	·5774	1·7321	·8660	60
31	·5150	·6009	1·6643	·8572	59
32	·5299	·6249	1·6003	·8480	58
33	·5446	·6494	1·5399	·8387	57
34	·5592	·6745	1·4826	·8290	56
35	·5736	·7002	1·4281	·8192	55
36	·5878	·7265	1·3764	·8090	54
37	·6018	·7536	1·3270	·7986	53
38	·6157	·7813	1·2799	·7880	52
39	·6293	·8098	1·2349	·7771	51
40	·6428	·8391	1·1918	·7660	50
41	·6561	·8693	1·1504	·7547	49
42	·6691	·9004	1·1106	·7431	48
43	·6820	·9325	1·0724	·7314	47
44	·6947	·9657	1·0355	·7193	46
45	·7071	1·0000	1·0000	·7071	45
	Cosine.	Co-tangent.	Tangent.	Sine.	Angle.

INDEX.

V

W

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